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Journal of Banking & Finance xxx (2005) xxx–xxx

Journal of BANKING & FINANCE

www.elsevier.com/locate/econbase

Is learning a dimension of risk?

Massimo Massa [a], Andrei Simonov [b,*]

[a] INSEAD
[b] Finance Department, Stockholm School of Economics, Box 6501, Stockholm 11383, Sweden

Received 13 January 2004; accepted 19 October 2004

Abstract

We empirically assess how the uncertainty induced by investors' learning about the fundamentals affects stock returns. We identify two components of induced uncertainty: learning uncertainty and dispersion of beliefs. We characterize these in terms of their relationship to uncertainty about the fundamentals as estimated by surveys of economic forecasters and to measures of uncertainty embedded in derivative markets (open interest and implied volatility). We show that both learning uncertainty and dispersion of beliefs are conditionally priced.

JEL classification: G11; G12; G14
Keywords: Conditional asset pricing; Time-varying risk factors; Learning uncertainty; Filtering

1. Introduction

The finance literature agrees on the existence of some "induced uncertainty", generated by investors' processing of information. This uncertainty arises both from the way in which investors update their beliefs about the fundamentals – *learning uncertainty* – and from the way these beliefs differ across investors – *dispersion of beliefs*. To date, little has been done to separate these two components and, in general, to

* Corresponding author. Tel.: +46 8 736 9159; fax: +46 8 312 327.
E-mail address: andrei.simonov@hhs.se (A. Simonov).

0378-4266/$ - see front matter
doi:10.1016/j.jbankfin.2004.10.001

empirically investigate the impact of induced uncertainty on stock returns. While it is well known that the process of learning affects investors, generates uncertainty and influences investor portfolio decisions, it is less well understood whether it aggregates across investors in such a way as to affect returns, or whether it simply averages out. One possible explanation of this fact lies in the standard framework hitherto used as a basis for empirical tests: the Efficient Markets Hypothesis (EMH). EMH places very strong restrictions on investors' learning. Investors start with arbitrary Gaussian priors at the beginning of history and then they rationally update them by using a Bayesian rule. All the history is assumed to be known and remembered.

These seemingly innocuous assumptions generate learning processes that quickly converge to the long-run stationary equilibrium. Moreover, in the standard case of normally distributed variables, this leads to a deterministic conditional variance. However, learning uncertainty – i.e., the conditional variance – is the very dimension of uncertainty forecasters use to gauge the reliability of their estimates and on which portfolio managers assess the degree of riskiness of their strategies. It is also the key variable used in business sensitivity analysis. Therefore, it is unfortunate that such a variable turns out being a merely deterministic one, without a dynamics that can directly impact stock returns. To cope with this problem the literature has resorted to two different approaches.

The first one is based on the relaxation of the assumption of Gaussian distributed priors. This is enough to generate a stochastic conditional variance (Detemple, 1991). In this case, "the posterior beliefs of the investor are characterized by two sets of sufficient statistics implementable in the form of a filter, (1) the vector of conditional means and (2) a set of sufficient statistics for the conditional variance–covariance matrix." An alternative approach posits that investors forget and regularly reassess their forecasts on the basis of a fresh set of priors and of a limited information set based on a fixed window of observations. This approach has a flavor analogous to the recent models of bounded rationality due to limited memory (Mullainathan, 1998). Individuals forget part of the history and use the same rational learning rule in the part they remember. [1] This approach is similar to the one brought forward by Bossaerts (1999) that relaxes the assumption of predetermined priors defined at the beginning of history and constantly updated. History is truncated into "windows" and the time series dimension is collapsed in a series of separate cross-sections. These take the form of (overlapping) histories characterized by "arbitrariness in the priors ... at the starting point" of each of the histories. While investors "may hold an arbitrary prior at the beginning of each history in the dataset, in the subsequent history, however, they are required to use all the available information to Bayesian update the priors, ... as in the EMH". That is, once the new priors are determined at the beginning of each history, then investors behave as fully rational agents. The advantage of this "near-rational" approach is that, while it

[1] In his model, the existence of forgetful individuals may lead to negative correlation between forecast errors over time.

"looses the natural connection [between the end of each period and the beginning of another], gains in robustness to certain deviation of the EMH."

Altering the learning window used by the investors to make forecasts provides a link with behavioral theories. Indeed, as shown by Brav and Heaton (2002), the behavioral bias of "representativeness" may be explained in terms of heavy weighting of more recent data with respect to old one.

We rely on this approach and construct two proxies that separately identify learning uncertainty and dispersion of beliefs. In particular, we construct a trade-based measure of uncertainty that mostly correlates with dispersion of beliefs and a price-based measure of uncertainty that mostly correlates with learning uncertainty. We then test whether this identification is correct, using outside measures of uncertainty and dispersion of beliefs. These measures are macro-founded (survey of professional forecasters) as well as micro-founded (dispersion of analysts' forecasts, open interest and implied volatility).

Once we have decomposed induced uncertainty into a part related to learning uncertainty and another related to dispersion of beliefs, we examine whether they are priced. To deal with the issue of time variation of the risk factors, we use a conditional pricing model. We show that both learning uncertainty and dispersion of beliefs are conditionally priced. They represent components of the risk premium that are separate from the fundamental one. We also find that learning uncertainty and dispersion of beliefs affect the time-variation of the economic risk premiums.

The remainder of the paper is structured as follows. In Section 2, we present a brief survey of the related literature. In Section 3, we lay out the hypotheses we test. In Section 4, we describe how we construct our proxies for induced uncertainty and how we identify them. We present the empirical results and report the main findings in Section 5. A brief conclusion follows.

2. Relation to existing literature

There is a growing theoretical literature that focuses on the impact of learning on optimal portfolio choice. For example, Bawa et al. (1979), Kandel et al. (1995), Kandel and Stambaugh (1996), and Stambaugh (1999) have thoroughly analyzed the effects of learning and estimation uncertainty on portfolio decisions. Brennan (1998) shows how estimation risk, that is the risk of "learning bad news", affects agents' incentive to invest in stocks. Barberis (2000) argues that the process of learning can tilt the portfolio composition so much that the need to hedge estimation risk may prevail over the need to hedge systematic uncertainty. Avramov (2002) presents evidence that model uncertainty is more important than estimation risk in portfolio choice.

This literature has also extensively investigated the impact of learning on prices (Dothan and Feldman, 1986; Gennotte, 1986; Wang, 1993; He and Wang, 1995; Brennan, 1998; Barberis et al., 1998; Veronesi, 2000; Brennan and Xia, 2001b). Xia (2000) calibrates a single agent economy and finds significant effects produced by agents' learning. Calibration to US data suggests that non-observability of expected dividend growth rate introduces an element of learning that increases the volatility of stock prices (Brennan and Xia, 2001a). Lewellen and Shanken (2002) show how parameter uncertainty is an important factor for characterizing and testing market efficiency. Timmermann (1993) shows, in a partial equilibrium framework, that systematic mistakes about future dividends are directly reflected in stock price volatility.

However, the *empirical* issue of the relevance of these effects on asset prices has, up to now, drawn little attention. Only recently, have Pastor and Veronesi (2003) investigated the impact of learning uncertainty about stock profitability to explain the market-to-book premium. This is all the more surprising as the empirical asset pricing literature has extensively focused on identifying state variables that are priced (Ferson and Harvey, 1991, 1993, 1997, 1999; Harvey, 1989, 1995; Bekaert and Harvey, 1995, 1997). Yet, none has tried to quantify the direct impact of learning on asset prices, assessing *whether it is actually priced*, and *how it relates to the time-variation of risk premia*.

Even more problematic has been the empirical analysis of how dispersion of beliefs affects returns. While volume has been largely considered as a good proxy for dispersion of beliefs and there is much convincing evidence of the *correlation* between trading volume and stock returns (Karpoff, 1987), *there is no evidence to support the claim that trading volume is a priced factor*. Indeed, trading volume is found to be a "characteristic", but not a risk factor (Chordia et al., 2001). Even the classic identification of volume with dispersion of beliefs has been put in doubt. Diether et al. (2002) and Ghysels and Juergens (2002) disregard volume and use the dispersion of analysts' earning forecast as a proxy for dispersion of beliefs. While they document the impact of dispersion of beliefs on stock returns, they do not relate it to volume, nor do they consider its time variation and its relation to learning uncertainty.

We take a different approach and rely on a "behavioral" extension of a standard dynamic rational expectation models with asymmetric information (Wang, 1993; He and Wang, 1995). We depart from rationality as investors, are "ignorant", in the sense that they apply the aforesaid sub-optimal learning rule, but they not aware of their limitation (Brandt et al., 2001). [2] Therefore, they do not correct their mistakes. This is close to the idea of "rational structural uncertainty" (Brav and Heaton, 2002). Investors, even if aware of the limitation of the learning rule, do not incorporate it into their decision for the adjustment costs that this would entail. From a theoretical perspective, Simon (1955), Marschak (1968), Einhorn (1970, 1971), and Payne et al. (1990) show how this behavior may be optimal if the computational costs are high. From a more empirical perspective, the standard practice of exponential smoothing with fixed rolling windows used in commercial forecasting provides evidence in this direction.

[2] Brandt et al. (2001) show that alternative learning behaviors alter significantly both the level and the time variation of the conditional moments of returns.

3. The hypotheses to be tested

We start with some notation. We define "learning uncertainty" the degree of uncertainty of investors' forecasts. It is meant to capture the uncertainty that arises from the fact that investors learn about the fundamental value of the asset. Our proxy is the conditional variance of a filtering process that uses stock returns to infer the state of the economy. In particular, let us consider the following Kalman filter:

$$\begin{cases} X_{t+1} = AX_t + u_t, \\ \mathbf{R}_t = \mathbf{B}\mathbf{X}_t + \mathbf{v}_t, \end{cases} \tag{1}$$

where $\mathbf{R}_t$ is the vector of returns of the assets and X_t is the underlying state which the returns are a function of. We can think of X_t as the main factor affecting the economy (e.g., GDP). $\mathbf{R}_t$ is the vector of portfolio returns constructed, in the spirit of Fama and French (1993), aggregating the stocks in either 25 value-weighted portfolios on the basis of the book-to-market and size classification, or 30 portfolios on the basis of four-digit SIC codes. The residuals u_t and v_t are Gaussian white noise sequences with $Var(u_t) = Q$ and $Var(\mathbf{v}_t) = \mathbf{V}$, respectively. Our measure of learning uncertainty is the investors' *conditional variance* of the underlying state (i.e., $\mathrm{LU}_t = Var[X_{t+1}|\{\mathbf{R}\}_{1,\ldots,t}]$).

A simple example will convey the intuition behind our proxy LU_t. Let us, for example, consider an investor who knows that stocks returns are a function of the state of the economy (i.e., GDP). He does not observe GDP and can only use the information contained in stock returns to infer it. Therefore, every morning, he observes stock returns ($\mathbf{R}_t$) and uses them to determine GDP (X_t) and forecast its value in the future. The future value of GDP will allow him determine the correct stock returns and invest accordingly. The system of Eq. (1) is meant to reproduce such a forecasting process. That is, at each point in time, the investor looks backward a certain number of days and uses the past stock returns to train his filter. The conditional variance (LU_t) represents the uncertainty about his forecast. The higher the variance, the less accurate the forecast is and the more uncertain the investor will feel. This higher uncertainty will be reflected in his portfolio choice and, in equilibrium, in the stock prices.

Our second measure of induced uncertainty tries to capture the dispersion of beliefs among investors. We define "dispersion of beliefs" the differences in the expected value that investors, characterized by different information sets, attribute to an asset or to the state variables that determine the value of the asset (e.g., future dividends).

We consider a rational expectation framework where investors are differentially informed. The fact that investors are not fully informed induces them to "learn". This learning generates uncertainty against which investors want to hedge. That is, investors attempt to protect themselves against the risk of "learning bad news: that the mean (return) is low" (Brennan, 1997). Higher learning risk prompts the investors to adjust their portfolios to hedge the increased risk. The fact that portfolio decisions are conditioned by this learning uncertainty implies that in equilibrium stock prices are affected by it and that learning uncertainty becomes an additional risk factor.

We make two assumptions that make us depart from the standard framework. First, we assume that investors "re-evaluate" their assessment of the state of the world by using a new set of priors. [3] The updating is a standard Kalman updating process. The second assumption is about limited memory. In the standard framework, the whole history matters and the statistics are a function of it. We, on the contrary, assume that investors update their estimates on the basis of a fixed rolling window of observations. This is consistent with models of bounded rationality due to limited memory (Mullainathan, 1998) as well as with models based on rational structural uncertainty (Brav and Heaton, 2002). We assume that investors "forget". In particular, at each time t, investors only consider the previous $t - N$ data points. As in Bossaerts (1999), history is truncated into a series of "windows" and a new set of priors is used at each point in time. These take the form of (overlapping) histories characterized by "arbitrariness in the priors ... at the starting point" of each of the histories. The joint effect of different priors, reassessment of the assumed underlying parameters and limited memory breaks the continuity of learning of the investors and, effectively, makes the conditional variances stochastic.

We assume information asymmetry and a nested asymmetric information structure – i.e., the informed investors have access to the same information of the less informed (i.e., prices) and to something else. This allows the more informed investors to condition their trading decisions also on the forecasts of the less informed and implies that the learning errors of the less informed become themselves additional "factors" that affect stock returns. We can summarize these two intuitions with the following claim.

Claim 1: If there is information asymmetry and information is nested, learning uncertainty and dispersion of beliefs affect both stock returns and trading volume.

This claim is based on an extension of Theorem 4.2, 4.3, and 4.4 of Wang (1993). The test of the restriction that dispersion of beliefs is priced is, in fact, a joint test of pricing and asymmetry of information.

The aforesaid discussion provides a direct way of constructing a proxy for learning uncertainty. Moreover, the dynamic nature of the relationship between trading

[3] This may happen in the case of a change in the composition of the investors in the economy. Let us assume, for example, that at the end of each trading period a new cohort of investors enters the economy with different priors. We may think of the new cohort as a change in the overall composition of the investors in the economy as well as a rotation of the class of marginal investors. The new investors will re-elaborate the same information contained in the observed signals (i.e., prices) by using their own priors.This approach would suggest that the different cohorts of investors would use the same rational learning (i.e., they have the same likelihood), but adopt different priors. This will lead to new beliefs that will be impounded in prices. This interpretation would also be consistent with the" differential interpretation hypothesis", that is with the idea that investors have different beliefs about the distribution of the signals (Kandel and Pearson, 1995; Kandel and Zilberfarb, 1999). If the new class of investors interprets the signals differently, their different beliefs will be impounded into prices.

Table 1
Descriptive statistics for stock portfolios

Size and book-to-market portfolios (25 portfolios)				Industry portfolios (30 portfolios)			
Portfolio	Mean (RET)	Median (RET)	Std. Dev. (RET)	Portfolio	Mean (RET)	Median (RET)	Std. Dev. (RET)
FF01 (BM1 SZ1)	−0.20	−0.01	8.07	Food	0.73	0.69	4.70
FF02 (BM2 SZ1)	0.49	0.77	7.01	Beer	0.75	0.67	5.84
FF03 (BM3 SZ1)	0.73	0.93	6.31	Smoke	1.06	1.08	5.77
FF04 (BM4 SZ1)	0.89	1.24	6.12	Games	0.78	0.92	7.23
FF05 (BM5 SZ1)	1.28	1.14	6.30	Books	0.65	0.67	5.76
FF06 (BM1 SZ2)	0.19	0.51	7.28	Cons Goods	0.59	0.70	5.01
FF07 (BM2 SZ2)	0.51	0.84	6.57	Apparel	0.66	0.43	7.56
FF08 (BM3 SZ2)	0.70	0.89	5.79	Health	0.76	0.73	5.28
FF09 (BM4 SZ2)	0.92	1.20	5.56	Chemicals	0.43	0.44	5.29
FF10 (BM5 SZ2)	1.16	1.49	5.85	Textiles	0.47	0.50	6.24
FF11 (BM1 SZ3)	0.24	0.48	6.45	Construction	0.50	0.30	5.80
FF12 (BM2 SZ3)	0.57	0.84	5.80	Steel	0.15	0.20	6.27
FF13 (BM3 SZ3)	0.59	0.86	5.23	Fabricated prod	0.31	0.52	5.91
FF14 (BM4 SZ3)	0.84	1.12	4.97	Electrical Eq	0.58	0.62	6.05
FF15 (BM5 SZ3)	1.02	1.25	5.85	Autos	0.42	0.10	5.87
FF16 (BM1 SZ4)	0.36	0.40	5.68	Carry	0.71	0.72	6.61
FF17 (BM2 SZ4)	0.47	0.66	5.34	Mines	0.30	0.08	7.26
FF18 (BM3 SZ4)	0.52	0.77	4.74	Coal	0.56	0.43	7.97
FF19 (BM4 SZ4)	0.81	1.08	4.96	Oil	0.54	0.60	5.19
FF20 (BM5 SZ4)	1.06	1.23	5.63	Util	0.42	0.37	3.90
FF21 (BM1 SZ5)	0.50	0.73	4.72	Telecom	0.54	0.55	4.31
FF22 (BM2 SZ5)	0.37	0.61	4.50	Services	0.78	0.86	6.74
FF23 (BM3 SZ5)	0.45	0.55	4.35	Business equipment	0.47	0.10	5.91
FF24 (BM4 SZ5)	0.63	0.87	4.48	Business supplies	0.45	0.49	5.17
FF25 (BM5 SZ5)	0.73	0.85	5.00	Transport	0.43	0.80	6.63
				Wholesale	0.77	0.79	6.53
				Retail	0.63	0.23	5.73
				Restaurants/hotels	0.73	1.07	6.95
				Finance	0.60	0.63	5.34
				Other	0.30	0.37	4.11

We report the descriptive statistics for the 25 size and book-to-market portfolios and for the 30 industry portfolios. The size and book-to-market portfolios are formed using the Fama and French (1993) procedure. The portfolios, which are constructed at the end of each June, are the intersections of five portfolios formed on size (market equity, ME) and five portfolios formed on the ratio of book equity to market equity (BE/ME). The size breakpoints for year t are the NYSE market equity quintiles at the end of June of t. BE/ME for June of year t is the book equity for the last fiscal year end in $t-1$ divided by ME for December of $t-1$. The BE/ME breakpoints are NYSE quintiles. Portfolios include NYSE and AMEX firms for 1964–1982 and NYSE, AMEX and NASDAQ firms for 1983–1998. To be included in the sample, the firm must have volume reported for given year. Descriptive statistics are compiled for June 1964–December 1998. We report statistics for the excess value-weighted returns RET (mean, median, and standard deviation, % per month).

volume, dispersion of beliefs and learning uncertainty suggests the type of pricing model to use for our empirical investigation. Indeed, if dispersion of beliefs changes over time, a *conditional pricing model* should better capture the covariation between returns and trading volume. We now move on to the construction of our measure of induced uncertainty.

4. Measuring induced uncertainty

Our goal is to construct measures of induced uncertainty that proxy for dispersion of beliefs and learning uncertainty and to relate them to stock returns. The main problem is the fact that the proxies that have been used in the literature are observed at too low a frequency (e.g., macro-based measures). Alternatively, their availability is limited in time (e.g., micro-based measures exploiting the information from the derivatives market). We will therefore proceed in two steps. First, we use financial data (i.e., returns and trading volume) to construct proxies for the two components of induced uncertainty. Then, we exploit already existing measures to verify the quality of our proxies. For this purpose, we will use macro- and micro-based measures of uncertainty.

4.1. A price-based measure of learning uncertainty

We construct LU_t by estimating the Kalman filter represented by the system of Eq. (1). We concentrate on the minimum information set *common to all the investors in the market*: observable stock returns. To determine the estimation window, we consider monthly intervals. In other words, we reestimate the Kalman filter at the end of each month on the basis of rolling windows made of the previous 500 trading days. This implies that investors' LU_ts at the end of the month are based on the observations of the previous 500 days, conditional on the new set of hypothesized parameters. Each month a new 500-day update is determined based on a new set of parameters. Time series of LU_ts are calculated for the period June 1964–December 1998. The size of the window implicitly defines the memory length of the investors. [4]

While LU_t is meant to capture learning uncertainty, it may, in fact, be proxying for stock market volatility. In order to ascertain this, we regress our measure of learning uncertainty on the monthly estimates of volatility of returns based on daily observations of CRSP indexes. We use both equally- and value-weighted indexes. The results display a very low correlation between the two variables (*Adjusted* R^2 is below 1.3%). This suggests that our *measure contains information different from that embedded in the standard measures of volatility.*

[4] Alternatively, we consider quarterly intervals. That is, we reestimate the Kalman filter at the end of each quarter, on the basis of *non-overlapping* rolling windows made of the previous three months' daily returns data. Given that the use of non-overlapping windows dramatically reduces the number of degrees of freedom, in this case, we use as input an aggregate index.

4.2. A trade-based measure of dispersion of beliefs

Our second measure of induced uncertainty tries to capture the dispersion of beliefs among investors. A possible proxy is the dispersion in analyst forecasts. However, these are available only from 1986. In the case of the standard deviation of macro-forecasts, instead, while available for a longer period, they come at a lower frequency (quarterly). We therefore consider a "trade-based" measure of uncertainty, – i.e., is based on the aggregate market trading volume or turnover. Trading volume is a direct function of both dispersion of beliefs and learning uncertainty. [5] Therefore, trading volume, once purged of the effects of learning and fundamental uncertainty, should be a good proxy for investors' dispersion of beliefs.

We proceed as follows. We first define, for each individual stock, turnover as the logarithm of the monthly ratio between total number of shares traded and the total number of shares outstanding. This measure, similar to the one used by Lo and Wang (2000) and Chordia et al. (2001), allows us to aggregate turnover across stocks of companies that are different in terms of size and number of share outstanding. [6]

Given that the two measures of induced uncertainty are related, we orthogonalize them by regressing turnover on LU_t and taking the residuals as our measure of trade-based uncertainty ($DISP_t$). This captures the component of turnover that is orthogonal to LU_t. The orthogonalization is done once for each sample we estimate (i.e., full sample and different sub-samples). Table 2, Panel A and B, report the descriptive statistics for the price-based measure, the trading-based measure and the components of the two measures orthogonal to three Fama and French (from here on "FF") factors. Table 2, Panel C, reports the autocorrelation coefficients up to 12th lag, Box–Ljung statistics and the results of Phillips–Perron unit root tests (with trend). Both variables appear to be stationary. [7] It is also interesting to note that LU_t is hardly related to any of the FF factors, while $DISP_t$ is positively related to HML.

[5] That is, trading volume is driven by investors' individual information sets and by their differences with respect to the common one. In other words, trading volume is a function of investors' learning uncertainty based on both their *private and common information sets*, while prices are a function of the learning uncertainty based only on investors' *common information set*.

[6] We experimented with different aggregations across stocks: logarithm of average monthly turnovers of the stocks part of the portfolio, the average of the logarithm of the monthly turnover of each single stock and the average monthly turnover of the stocks in the portfolio. Given that the results in the three specifications agree, we report only the ones based on the logarithm of average monthly turnovers of individual stocks. The other specifications are available upon requests from the authors. Moreover, to account for the fact that the volume for NASDAQ-listed stocks reported in CRSP is overstated due to double counting of the transactions between NASDAQ market-makers, we multiply volume for NASDAQ stocks by 0.56 (Atkins and Dyl, 1997).

[7] The tests of stationarity without trend and the augumented Dickey–Fuller test are available upon request from the authors. The results agree with those reported. It is interesting to note that on the level of individual firms, the measures of trade are often found to be non-stationary (Lo and Wang, 2000).

Table 2
Measures of induced uncertainty

	Lu		DISP	
Variable	Coefficient	*t*-Statistic	Coefficient	*t*-Statistic
Panel A: Induced uncertainty and FF factors				
Intercept	1.28	(43.12)	−0.25	(−0.31)
MKT_RF	−1.48	(−1.41)	4.91	(0.15)
SMB	−0.87	(−0.70)	−13.05	(−0.37)
HML	−0.53	(−0.69)	59.74	(2.56)
Adj. R^2	0.001		0.024	

Variable	Mean	Median	Std. Dev.	Minimum	Maximum	Skewness	Kurtosis
Panel B: Descriptive statistics of induced uncertainty							
LU	1.27	1.13	0.58	0.44	3.24	0.62	−0.41
DISP	0.00	−1.44	16.11	−27.85	52.77	0.57	−0.13
LU^0	0.00	−0.16	0.58	−0.88	1.98	0.63	−0.37
$DISP^0$	0.00	−2.20	15.85	−34.55	55.64	0.62	0.05

Lags	LU	DISP	LU^0	$DISP^0$
Panel C: Autocorrelation structure				
1	0.22	0.88	0.22	0.86
2	0.25	0.83	0.25	0.81
3	0.12	0.79	0.11	0.78
4	0.13	0.74	0.12	0.73
5	0.16	0.74	0.16	0.74
6	0.18	0.72	0.17	0.72
7	0.22	0.70	0.21	0.71
8	0.19	0.69	0.18	0.69
9	0.07	0.68	0.08	0.68
10	0.12	0.66	0.12	0.67
11	0.10	0.65	0.10	0.64
12	0.05	0.65	0.05	0.65
Box–Ljung statistics				
Q (24)	141	3479	132	3429
$p < \chi^2$	<0.0001	<0.0001	<0.0001	<0.0001
Phillips–Perron unit root test (*with trend*)				
Z_ρ	−375	−72	−375	−90
$p < Z_\rho$	<0.0001	<0.0001	<0.0001	<0.0001

Panel A reports the results of the regressions of our measures of induced uncertainty (LU and DISP) on the Fama and French factors. We use the Newey–West estimator with correction for autocorrelation. The residuals are the orthogonalized factors (LU^0 and $DISP^0$ respectively). In Panel B, we report the descriptive statistics of the LU and DISP and their components orthogonal to factors (LU^0 and $DISP^0$). Panel C reports the autocorrelation coefficients up to 12th lag, the Box–Ljung statistics and the results of Phillips–Perron unit root test (with trend). The lags are months. All the estimations are done for the period 1964:06–1998:12.

Table 3
Induced uncertainty and surveys

Variable	Start Date	Mean	Std. Dev.	Min	Max	Median number of responses per survey	Correlation between 6- and 12-month variable
Panel A: Standard deviation of macro-economic forecasts: Livingston Survey (descriptive statistics)							
RTL6	1959	0.050	0.183	0.009	1.521	38	0.191
RTL12	1959	0.037	0.011	0.014	0.088	38	
AUTO6	1966	0.060	0.025	0.025	0.140	47	0.935
AUTO12	1966	0.071	0.028	0.029	0.165	47	
IP6	1946	0.018	0.006	0.007	0.030	52	0.488
IP12	1946	0.027	0.012	0.010	0.097	51	
GDPX6	1946	0.013	0.005	0.006	0.024	55	0.838
GDPX12	1946	0.019	0.006	0.009	0.030	54	
UNP6	1961	0.053	0.018	0.024	0.106	53	0.847
UNP12	1961	0.076	0.024	0.037	0.136	53	
WAGE6	1949	0.017	0.013	0.008	0.114	36	0.266
WAGE12	1949	0.023	0.007	0.012	0.051	36	
CPROF6	1971	0.082	0.037	0.025	0.161	46	0.898
CPROF12	1971	0.102	0.041	0.039	0.195	45	

Forecast	Mean	Std. Dev.	Min	Max	Median number of responses per survey	Correlations			
						2 Quarters ahead	3 Quarters ahead	4 Quarters ahead	5 Quarters ahead
Panel B: Probability of recession: ASA-NBER Survey									
Mean probability									
1 Quarter ahead	20.49	23.99	0.52	88.71	34	0.88	0.62	0.26	−0.17
2 Quarters ahead	20.34	17.21	2.16	74.06	34		0.87	0.50	−0.01
3 Quarters ahead	19.16	11.72	4.03	58.80	34			0.79	0.29
4 Quarters ahead	18.13	7.18	4.56	36.31	34				0.78
Standard deviation (σ_π)									
1 Quarter ahead	19.30	13.24	2.47	55.25	34	0.73	0.44	0.21	0.05
2 Quarters ahead	19.60	13.24	1.19	56.44	34		0.75	0.42	0.19
3 Quarters ahead	18.81	13.61	0.93	56.76	34			0.71	0.45
4 Quarters ahead	18.09	13.81	0.72	68.26	34				0.81
Index of uncertainty $I_\pi = \pi(1-\pi)$									
1 Quarter ahead	0.106	0.073	0.001	0.250	34	0.82	0.53	0.20	−0.11

(*continued on next page*)

Table 3 (*continued*)

Forecast	Mean	Std. Dev.	Min	Max	Median number of responses per survey	2 Quarters ahead	3 Quarters ahead	4 Quarters ahead	5 Quarters ahead
2 Quarters ahead	0.133	0.067	0.021	0.250	34		0.82	0.45	0.04
3 Quarters ahead	0.141	0.055	0.039	0.250	34			0.79	0.37
4 Quarters ahead	0.143	0.043	0.044	0.231	34				0.82

	Model 1		Model 2		Model 3		Model 4	
Variable	Estimate	*t*-stat.	Estimate	*t*-stat.	Estimate	*t*-stat.	Estimate	*t*-stat.
Panel C: Induced uncertainty and dispersion of forecasts (the Livingston Survey)								
LU								
Intercept	0.61	(2.02)	0.45	(1.51)	0.90	(3.25)	0.71	(2.17)
RTL6	0.49	(2.92)	0.44	(1.99)	0.56	(3.43)	0.51	(2.41)
RTL12	5.58	(0.95)	6.02	(1.02)	5.58	(0.95)	6.02	(1.02)
AUTO6	−23.08	(−3.86)	−27.19	(−4.82)	−0.41	(−0.14)	−2.56	(−0.76)
AUTO12	22.02	(3.78)	23.91	(4.24)	22.02	(3.78)	23.91	(4.24)
IP6	23.07	(1.56)	27.69	(1.66)	20.79	(1.58)	22.84	(1.58)
IP12	−2.27	(−0.55)	−4.83	(−1.08)	−2.26	(−0.55)	−4.82	(−1.08)
GDPX6	18.48	(0.65)	34.96	(1.17)	−12.58	(−0.95)	−12.12	(−0.60)
GDPX12	−29.78	(−1.01)	−45.16	(−1.05)	−29.78	(−1.01)	−45.16	(−1.05)
UNMP6	−7.57	(−0.85)	−6.29	(−0.54)	6.24	(1.42)	11.57	(2.86)
UNMP12	12.40	(1.94)	16.03	(1.76)	12.40	(1.94)	16.03	(1.76)
WAGE6	−6.49	(−2.67)	−8.48	(−4.67)	−7.52	(−2.84)	−9.57	(−4.16)
WAGE12	−7.20	(−0.78)	−7.62	(−0.87)	−7.20	(−0.78)	−7.62	(−0.87)
CPROF6			−0.03	(−0.01)			0.03	(0.02)
CPROF12			1.07	(0.22)			1.06	(0.22)
R^2	0.293		0.359		0.293		0.359	
Adj. R^2	0.127		0.135		0.127		0.135	
PKK Stat.	4.31*		6.75*		3.92*		5.45*	
Break date	1979:Q4		1981:Q2		1981:Q2		1980:Q4	
DISP								
Intercept	44.32	(5.50)	44.09	(4.93)	26.56	(3.32)	30.01	(3.09)
RTL6	−5.53	(−1.75)	−6.18	(−1.70)	−5.47	(−1.54)	−5.83	(−1.50)
RTL12	4.93	(0.03)	30.24	(0.17)	4.93	(0.03)	30.24	(0.17)
AUTO6	259.32	(2.03)	315.73	(2.34)	80.81	(1.22)	107.87	(1.28)
AUTO12	−173.34	(−1.44)	−201.85	(−1.62)	−173.34	(−1.44)	−201.85	(−1.62)
IP6	−457.10	(−1.66)	−512.69	(−1.58)	−857.15	(−3.05)	−850.98	(−2.67)
IP12	−398.14	(−6.36)	−336.67	(−5.14)	−398.14	(−6.36)	−336.67	(−5.14)
GDPX6	−438.14	(−0.98)	−468.59	(−0.93)	−959.74	(−3.13)s	−815.71	(−1.92)
GDPX12	−500.17	(−1.50)	−332.86	(−0.73)	−500.17	(−1.50)	−332.86	(−0.73)
UNMP6	25.00	(0.15)	−99.43	(−0.51)	−64.75	(−0.65)	−147.68	(−1.28)
UNMP12	−80.53	(−0.76)	−43.30	(−0.33)	−80.53	(−0.76)	−43.30	(−0.33)
WAGE6	92.46	(1.92)	98.11	(1.77)	34.10	(0.62)	45.57	(0.82)
WAGE12	−408.42	(−1.89)	−367.67	(−1.69)	−408.42	(−1.89)	−367.67	(−1.69)

Table 3 (*continued*)

CPROF6		127.04	(1.08)		127.04	(0.36)
CPROF12		−133.95	(−1.32)		−133.95	(−1.32)
R^2	0.535	0.558		0.535	0.558	
Adj. R^2	0.426	0.403		0.426	0.403	
PKK Stat.	4.11*	5.14*		3.84*	6.36*	
Break date	1980:Q2	1981:Q2		1982:Q4	1981:Q4	

	Model 1		Model 2		Model 3	
Variable	Coefficient	*t*-stat.	Coefficient	*t*-stat.	Coefficient	*t*-stat.
Panel D: Induced uncertainty and dispersion of forecasts (*the ASA-NBER Survey*)						
LU						
Intercept	1.16	(7.19)	1.00	(5.79)	1.05	(6.02)
$I_{\pi1}$	−0.31	(−0.24)	0.01	(0.01)		
$I_{\pi2}$					1.30	(1.82)
$I_{\pi3}$	2.45	(1.94)				
$I_{\pi4}$			3.99	(2.70)	3.46	(2.53)
$\sigma_{\pi1}$	1.25	(0.65)	1.21	(0.63)		
$\sigma_{\pi2}$					−1.95	(−0.93)
$\sigma_{\pi3}$	−2.87	(−1.22)				
$\sigma_{\pi4}$			−3.64	(−1.19)	−1.84	(−1.10)
R^2	0.033		0.059		0.058	
Adj. R^2	0.001		0.026		0.025	
PKK Stat.	1.25		1.23		1.09	
Break date	1982:Q1		1979:Q3		1979:Q3	
DISP						
Intercept	17.44	(1.93)	13.85	(1.53)	14.46	(1.62)
$I_{\pi1}$	−48.99	(−1.50)	−38.12	(−1.07)		
$I_{\pi2}$					−34.14	(−0.76)
$I_{\pi3}$	88.28	(1.66)				
$I_{\pi4}$			108.75	(1.64)	125.61	(1.76)
$\sigma_{\pi1}$	49.37	(0.99)	19.89	(0.47)		
$\sigma_{\pi2}$					−36.64	(−0.59)
$\sigma_{\pi3}$	−270.46	(−2.71)				
$\sigma_{\pi4}$			−242.34	(−2.22)	−208.77	(−1.95)
R^2	0.150		0.135		0.153	
Adj. R^2	0.120		0.105		0.124	
PKK Stat.	4.39*		4.47*		4.67*	
Break date	1982:Q2		1982:Q3		1982:Q2	

Panel A reports the descriptive statistics of the standard deviations of the macro-economic forecasts of the Livingston Survey, calculated across forecasters. The survey data are from the Federal Reserve Bank of Philadelphia. We report the following variables: RTL, retail trade, AUTO, auto sales, both domestic and foreign, IP, industrial production index, GDPX, gross domestic product (prior to 1992 gross national product), UNP, civilian unemployment rate, WAGE, average weekly earnings in manufacturing, CPROF, corporate profit after taxes. We consider the 6- and 12-month ahead forecasts. To make them homogenous we use the growth rate, defined as a percentage change with respect to the previous month. For each variable we calculate the standard deviation across forecasts (i.e., responses in the survey per period). For each variable we report the year the variable was introduced in the survey, the descriptive statistics of our measure of dispersion of forecasts, the median number of responses per survey and the correlation between6- and 12-month ahead forecasts. Panel B reports the descriptive statistics for the mean and the standard deviation (σ_{π}) of the probability of the recession as well as the descriptive statistics of the index of

(*continued on next page*)

Table 3 (*continued*)

uncertainty ($I_\pi = (1-\pi)\pi$) as estimated by the participants of the ASA-NBER Survey of Professional Forecasters. The mean probability is calculated as the average of the probabilities of recession reported by the polled forecasters. The standard deviation (σ_π) is the standard deviation of the probabilities of recession across the polled forecasters. Both mean and standard deviation are expressed in percentage terms. To build the index of uncertainty, we first construct a measure of uncertainty for each forecaster (i.e., $(1-\pi)\pi$) and then we average it out across forecasters. Panel C reports the results of the regression of induced uncertainty on the dispersions of forecasts in the Livingston Survey. In models 1 and 2 we use the original variables of the Livingston Survey. In models 3 and 4 the 12-month ahead forecasts are first orthogonalized by taking the residuals of the regressions of 12-month forecasts on the 6-month forecasts. In models 1 and 3 we use data from December 1966 (when forecasts of auto sales were added to the survey) to December 1998 (total 65 observations). In models 2 and 4 we use the data from June 1971 (date of addition of corporate profitability forecast) to December 1998. Estimations are done by using the Newey–West methodology with correction for autocorrelation up to 4 lags. *t*-Values are reported in parentheses. We also report the result of a Ploberger et al. (1989) (PKK) test of structural stability. A "*" signals that the PKK statistics rejects the null at the 5% level. Panel D reports the results of the regression of our proxies for induced uncertainty on the measures of dispersions of forecasts in the ASA-NBER Survey. Alternative specifications are estimated with different sets of variables.

4.3. An identification of learning uncertainty and dispersion of beliefs

We now have available two measures of uncertainty: one price-based and the other trade-based. Our claim is that the first proxies for learning uncertainty, while the second proxies for dispersion of beliefs. In order to prove this, we make use of measures of uncertainty that can be derived either from macro-economic surveys of economists and professional forecasters, or from micro-based variables. [8]

4.4. A macro-economic surveys

We consider the Livingston Survey and the ASA-NBER Survey. The Livingston Survey was started in 1946 by the late economist Joseph Livingston and is the oldest continuous survey of economists' expectations. It summarizes the forecasts of economists from industry, government, banking, and academia. The Federal Reserve Bank of Philadelphia took over the responsibility for the survey in 1990. Every June and December, survey participants are asked to forecast a set of key macro-economic variables for the end of the current month, two and four quarters ahead. [9] The number of participants varies over time, from 23 to 67 with the median around 50. [10] Descriptive statistics of the Livingston Survey are reported in Table 3, Panel A.

We select the variables that have the longest and most detailed coverage in terms of the numbers of forecasts: retail sales (RTL), auto sales (AUTO), industrial production (IP), GDP (GDPX), civilian unemployment rate (UNMP), average weekly

[8] For a review of available sources see Croushore (1993) and references therein.

[9] The full list can be found at http://www.phil.frb.org/econ/liv/.

[10] The data of the Survey have been used extensively in the financial literature. For example, Lakonishok (1980) and Yoon and Edelstein (1989) studied biases in the Livingston Survey, and Gultekin (1983) focused on the influence of inflation forecast on returns.

earnings in manufacturing (WAGE), corporate profit after taxes (CPROF). For each of these variables, we construct the standard deviations of the forecast values. [11]

The ASA-NBER Survey of Professional Forecasters was initiated in 1968 and generates the estimated probability of recession (π) from 1 to 4 quarters ahead. It has a quarterly frequency. Descriptive statistics of it are displayed in Table 3, Panel B, where we report the mean and standard deviation of the probability of recession as estimated by participants of the ASA-NBER Survey. We derive two measures of uncertainty from it. The first is constructed as the standard deviation of the forecasts of the respondents to the Survey (σ_π). Given that this measure proxies for dispersion of uncertainty across investors, we expect it to be more closely related to the dispersion of beliefs. A second measure is the average of the uncertainty of the respondents (I_π). The uncertainty is constructed as $\pi(1-\pi)$, where π is the probability of recession. This measure peaks at 0.25 when the respondent is more uncertain ($\pi = 0.5$) and goes to zero when he is certain ($\pi = 0$ or 1). Given that this measure is more related to the uncertainty as perceived by each respondent, we think that it provides a closer proxy for learning uncertainty. The joint use of these two measures allows us to identify the two components of induced uncertainty.

We regress our measures of induced uncertainty on these survey-based measures of uncertainty. In particular, in the case of the Livingston Survey, the explanatory variable is the dispersion of forecasts, while in the case of the ASA-NBER Survey, the explanatory variables contain *both* the dispersion of forecasts (σ_π) and the measures of pure uncertainty (I_π). We expect both our measures of induced uncertainty to be related to the dispersion of forecasts derived from the Livingston Survey, as this is related to both dispersion as well as uncertainty. However, in the case of the ASA-NBER Survey, where we have separate proxies for dispersion and uncertainty, the price-based measure should be mostly related to I_π, while the trade-based measure should be mostly related to σ_π.

We report the results in Table 3, Panel C, for the Livingston Survey, and Panel D for the ASA-NBER Survey. Given that the forecasts on corporate profits (CPROF) are available for a shorter period, we consider a specification inclusive of them and one where they are omitted. The results show that both measures of induced uncertainty correlate with the dispersion of the forecasts contained in the Livingston Survey. The correlation is significant for both components of induced uncertainty. The *Adjusted* R^2 is around 13% in the case of the price-based uncertainty and 43% in the case of the trade-based uncertainty. In particular, the price-based measure is mostly related to the dispersion of the six-month ahead forecasts of retail, auto sales and average weekly earnings in manufacturing and to the 12-month ahead forecasts of auto sales. The trade-based measure correlates mostly to the dispersion of six-month

[11] To make these forecasts homogenous, we first calculate the growth rate of the forecast values with respect to the last reported ones and then calculate their standard deviations. Also, in order to control for the strong collinearity between 6- and 12-month ahead forecasts, we also used the approach of Yoon and Edelstein (1989) and orthogonalize the 12-month ahead forecast on the 6-month ahead one. The residuals define the new variable.

ahead forecasts of auto sales and average weekly earnings in manufacturing and to the 12-month ahead forecasts of industrial production.

The picture changes when we consider the measures of uncertainty based on the ASA-NBER Survey. Here, we can run a horse race between dispersion and pure uncertainty. Given that forecasts for different quarters are correlated, we consider alternative specifications with different forecasts. The results agree with our identifying restriction. The price-based measure of uncertainty is only related to the proxy for pure uncertainty (I_π), while it is never related to the proxy for dispersion of forecasts (σ_π). The opposite is true for trade-based uncertainty that is related to the proxy for dispersion of forecasts (σ_π), while it is never related to the proxy for pure uncertainty (I_π). These results give a first indication that induced uncertainty is related to fundamental uncertainty and actually seems to lead it. Also, if we decompose it into the two components, it appears that the price-based measure of uncertainty is related mostly to pure uncertainty or risk, while the trade-based one is related mostly to dispersion of beliefs.

4.5. *A micro-based approach*

Let us now investigate this issue in more detail by focusing exclusively on financial data. To find proxies for the dispersion of beliefs and for risk that can be directly derived from financial data is a difficult task – especially for the entire 1968–1998 period. Indeed, the only proxies the literature has identified are open interest and implied volatility: the former as a proxy for dispersion of beliefs (Bessembinder et al., 1996; Christensen and Prabhala, 1998) and the latter as a proxy for risk. However, information on such variables is not available for the entire period under consideration. We will therefore focus only on the sub-sample in which these variables are available, and estimate:

$$E_t = \alpha + \beta \mathrm{OI}_t + \gamma \mathrm{IV}_t + \boldsymbol{\delta}\mathbf{C}_t + \varepsilon_t, \tag{2}$$

where E_t is our measure of induced uncertainty (i.e., LU_t or DISP_t) and OI_t and IV_t are, respectively, open interest and implied volatility on all financial and stock index futures and options traded on CBOT. $\mathbf{C}_t$ represents a set of control variables. [12]

The results, reported in Table 4, show that price-based uncertainty is related *only* to implied volatility and trade-based uncertainty *only* to open interest. These results are very robust and hold in all the tested specifications. The *Adjusted* R^2 is high (12% for the case of the price-based measure of uncertainty and 53% for the case of the trade-based measure of uncertainty) even in the case where no control variables have been included. The implication is that trade-based uncertainty is related mostly to dispersion of beliefs, while price-based uncertainty is related mostly to risk. This is consistent with the results in the previous section.

As additional robustness check, we also consider a measure of dispersion based on analysts' forecasts. This variable (IBESDISP) is the average dispersion of next fiscal quarter EPS forecast (scaled by mean EPS) as reported by IBES on the last day of the month. [13] We estimate how our measures (LU and DISP) are related to it. LU is not related to IBESDISP, while DISP is strongly related to it. This supports our

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Table 4
Induced uncertainty and financial uncertainty

	LU				DISP			
Variable	Coefficient	*t*-stat.	Coefficient	*t*-stat.	Coefficient	*t*-stat.	Coefficient	*t*-stat.
Intercept	0.73	(3.25)	0.72	(3.06)	−7.06	(−1.19)	−7.07	(−1.36)
IV	0.03	(6.23)	0.03	(5.95)	−0.13	(−0.61)	−0.07	(−0.49)
OI	0.01	(0.13)	0.03	(1.46)	15.79	(5.83)	12.04	(5.77)
IBESDISP			−0.03	(−0.44)			129.5	(3.74)
R^2	0.122		0.124		0.538		0.552	
Adj. R^2	0.111		0.108		0.532		0.544	

This table reports the results of the regression of our proxies for induced uncertainty on implied volatility (IV) and open interest (OI). IV is the implied volatility index and OI is the open interest on all the financial and stock index futures and options traded on the CBOT. IBESDISP is average standard deviation of next fiscal quarter EPS forecast (scaled by mean EPS) as reported by IBES as of last day of the month. Only observations for firms with more than two analysts were considered. Other variables are defined as in Table 4. The estimation period is 1986:01–1998:12. All the estimates for DISP are multiplied by 100. The coefficient on OI has been multiplied by 10^6.

working assumption that our trade-based measure is proxying for dispersion of beliefs.

5. Is induced uncertainty priced?

We are now ready to study whether both measures of uncertainties are priced. Given that we have ascertained in the previous section that these two measures of induced uncertainty proxy for different dimension of uncertainty, from now on, we will call learning uncertainty the price-based measure of uncertainty and dispersion of beliefs the trade-based one. We consider a conditional specification. Let us see it in more detail. We assume that there are M portfolios of stocks, with $j = 1, \ldots, M$ and N factors that affect returns, with $n = 1, \ldots, N$. The factors (F_n) include the standard ones (i.e., the three Fama and French factors and the Carhart momentum factor) and our measures of induced uncertainty (i.e., LU and DISP). The standard factors are derived from K. French's web page. The test is based on the conditional framework laid out by Dumas and Solnik (1995). We outline here the main restrictions of this approach, referring to their paper for further details:

$$E[R_{j,t}|\Omega_{t-1}] = \sum_{n=1}^{N} \lambda_{n,t-1} \mathrm{cov}[R_{j,t}, F_{n,t}|\Omega_{t-1}], \tag{3}$$

where $R_{j,t}$ is the return on the jth risky portfolio in excess of the riskless rate from time $t-1$ to t, $F_{n,t}$ represents the nth factor (or source of risk) for the same period, $\lambda_{n,t-1}$ is the price of the nth source of risk and Ω_{t-1} is the information set investors use in order to make their portfolio decision. Let us see how Eq. (3) generates a set of testable restrictions. We define m_t as the marginal rate of substitution between returns at date t and at date $t-1$ and $R_{f,\ t-1}$, as the conditionally risk-free rate for the period starting at $t-1$. Standard theory (Cochrane, 2001) tells us that the first order conditions of the portfolio choice problem are:

$$\begin{cases} E[m_t(1+R_{f,t-1})|\Omega_{t-1}] = 1, \\ E[m_t R_{j,t}|\Omega_{t-1}] = 0. \end{cases} \tag{4}$$

By using Eqs. (3) and (4), Dumas and Solnik (1995) show that m_t can to be defined as

$$m_t = \left[1 - \lambda_{o,t-1} - \sum_{n=1}^{N} \lambda_{n,t-1} F_{n,t}\right] / \left(1 + R_{f,t-1}\right), \tag{5}$$

where $\lambda_{o,t-1}$ appears as a "way of ensuring that the system of Eq. (4) is satisfied." The system of Eq. (4) defines the orthogonality conditions of the econometric specification.

We assume that the information set Ω_{t-1} is made of a vector of instrumental variables $\mathbf{Z}_{t-1}$. These variables were used by Ferson and Harvey (1991, 1999), Dumas and Solnik (1995). They include: a constant, the junk premium spread between Moody's Baa and Aaa corporate bond yields (JUNK), the dividend yield of the S&P 500 index (DIVY), the term premium spread between a 10-years and a one-year Treasury bond yield (TERM), the spread between the three-month and one-month T-bill (HB3), January dummies, the return of the market portfolio (R_{MKT}) and the lagged one-month T-bill yield (TB30D). We also assume a linear relationship between such state variables and the λs, such that: $\lambda_{o,t-1} = -\mathbf{Z}'_{t-1}\boldsymbol{\delta}$ and $\lambda_{n,t-1} = \mathbf{Z}'_{t-1}\boldsymbol{\phi}_n$, where $\boldsymbol{\delta}$ and $\boldsymbol{\phi}$s are time-invariant vectors of weights. In particular, $\boldsymbol{\delta} = [\delta_c, \delta_J, \delta_D, \delta_T, \delta_H, \delta_{Ja}, \delta_M, \delta_Y]'$, where δ_c, δ_J, δ_D, δ_T, δ_H, δ_{Ja}, δ_M, δ_Y are the sensitivities of the intercept term ($\lambda_{o,t-1}$) to the information variables in the order they have been previously defined. Analogously, $\boldsymbol{\phi} = [\phi_c, \phi_J, \phi_D, \phi_T, \phi_H, \phi_{Ja}, \phi_M, \phi_Y]'$, where ϕ_c, ϕ_J, ϕ_D, ϕ_T, ϕ_H, ϕ_{Ja}, ϕ_M, ϕ_Y are the sensitivity of the price of the nth source of risk ($\lambda_{n,\ t-1}$) to the information variables in the order they have been previously defined. We define the innovation ξ_t in the marginal rate of substitution as

$$\xi_t = 1 - m_t(1 + R_{f,t-1}). \tag{6}$$

Using Eq. (5), we can write:

$$\xi_t = -\mathbf{Z}'_{t-1}\boldsymbol{\delta} + \sum_{n=1}^{N} \mathbf{Z}'_{t-1}\boldsymbol{\phi}_n F_{n,t}. \tag{7}$$

Let us define $h_{j,t} = R_{j,t} - R_{j,t}\xi_t$. The pricing conditions of Eq. (4) imply the following orthogonality conditions:

$$\begin{cases} E[\xi_t|\Omega_{t-1}] = 0 \\ E[\mathbf{h}_t|\Omega_{t-1}] = 0. \end{cases} \tag{8}$$

where $\mathbf{h}_t$ is the vector that contains the $h_{j,t}$ stacked for all the considered portfolios. Defining the vector of residuals $\boldsymbol{\varepsilon}_t = (\xi_t, \mathbf{h}_t)$, and combining the orthogonality conditions, we have

$$E[\boldsymbol{\varepsilon}_t|\mathbf{Z}_{t-1}] = 0, \tag{9}$$

which implies the restriction $E[\boldsymbol{\varepsilon}_t'\mathbf{Z}_{t-1}] = 0$, whose sample version is

$$\mathbf{Z}'\boldsymbol{\varepsilon} = 0. \tag{10}$$

Here $\mathbf{Z}$ is TxQ matrix, where Q is the number of instruments that are employed and $\boldsymbol{\varepsilon}$ is $T \times (1 + M)$ matrix, where T is the number of observations over time and M is the number of portfolios we use (25 in the book-to-market and size specification and 30 in the industry specification). This represents a total of $Q \times (1 + M)$ conditions. The moments conditions of Eq. (10) represent our testable restrictions. Notice that λs have now been replaced by combinations of instruments ($\mathbf{Z}$), such that $\lambda_{o,t-1} = -\mathbf{Z}_{t-1}'\boldsymbol{\delta}$ and $\lambda_{n,t-1} = \mathbf{Z}_{t-1}'\boldsymbol{\phi}_n$, where $\boldsymbol{\delta}$ and $\boldsymbol{\phi}$s. This allows us to estimate $\boldsymbol{\delta}$ and $\boldsymbol{\phi}$ directly, without estimating the λs. The intuition behind the restrictions that link λs to $\mathbf{Z}$ is that risk premia (i.e., λs) can be predicted using a set of observable variables. Rewritten this way, the estimation of Eq. (10) is just the estimation of a series of orthogonal restrictions with robust standard errors, performed using standard GMM procedures. By minimizing the average deviation from these moment conditions, we derive the best estimates of the parameters $\boldsymbol{\delta}$ and $\boldsymbol{\phi}$. The deviations in the various moment conditions are weighted by a weighting matrix, W, which is the inverse of a consistent covariance matrix of sample moment conditions. Given that the main variables of interest (LU and DISP) have been constructed as a by-product of a previous estimation, the GMM estimates may be subject to a generated regression problem. To address this issue, we use the parametric bootstrap method developed by Smith and McAleer (1993). In Tables 5 and 6, next to the ordinary statistics, we also report the value of the bootstrapped statistics.

We will test whether the factors are priced by using the test of over-identifying restrictions for Eq. (10). That is, under the null that the system (8) hold, the value of the quadratic form of Eq. (10) is asymptotically distributed as a χ^2 with degrees of freedom equal to the number of orthogonality conditions minus the number of parameters (Dumas and Solnik, 1995).

We perform our analysis on portfolios. Following the Fama and French approach, we aggregate the stocks in either 25 value-weighted portfolios on the basis of the book-to-market and size classification, or 30 portfolios on the basis of four-digit SIC codes. We differ from the standard procedure in that we limit our universe of securities by using only securities that have volume reported. This effectively reduces our sample during the 1963–1982 to NYSE/AMEX stocks. Extensive robustness checks (available upon request) show that the results are not sensitive to the subtraction of NASDAQ firms from our sample. The summary statistics of the 25 size/book-to market portfolios and 30 industry portfolios are reported in Table 1. We can see that even if the sample is different (we used the period 1963–1998, while Fama and French (1993) used the period 1963–1990) the main statistics are similar to the Fama and French ones.

Following the standard methodology, we construct the additional factors as the one-period ahead change of our measures of informational uncertainty. The basic factors are the Fama and French factors (Market, HML and SMB) augmented by

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Table 5
Conditional tests of pricing

	δ	ϕ_{MKT}	ϕ_{HML}	ϕ_{SMB}	ϕ_{UMD}	ϕ_{LU}	ϕ_{DISP}
Panel A: 25 book-to-market and size portfolios							
Constant	0.42	−0.03	5.83	−7.23	−0.02	15.65	0.00
	(5.76)***	(−0.02)	(1.99)	(−2.87)*	(−0.77)	(2.36)	(0.29)
JUNK	103.31	3175.76	−2519.57	−5086.30	−51.55	7395.14	−52.51
	(1.31)	(1.83)	(−0.91)	(−1.65)	(−2.07)	(1.11)	(−3.83)***
DIVY	−72.92	1082.00	−119.12	5477.28	30.65	−5019.34	2.94
	(−1.76)	(1.22)	(−0.08)	(3.73)**	(2.09)	(−1.83)	(0.53)
TERM	3.91	52.78	21.99	−109.09	−1.55	−162.42	0.08
	(3.54)**	(2.38)	(0.51)	(−3.56)**	(−4.86)***	(−1.68)	(0.71)
HB3	21.04	583.27	−283.80	−75.79	8.31	−571.80	−0.42
	(3.23)**	(4.64)***	(−1.05)	(−0.27)	(4.68)***	(−0.89)	(−0.39)
$\mathrm{DUMMY}_{\mathrm{January}}$	0.17	−0.14	4.56	−4.15	0.01	−11.70	0.00
	(2.72)	(−0.14)	(2.03)	(−2.74)**	(0.50)	(−2.18)*	(0.45)
R_{MKT}	−0.81	−11.90	7.89	153.10	0.34	71.97	0.12
	(−1.30)	(−1.18)	(0.60)	(10.15)***	(2.54)*	(2.55)**	(3.14)**
TB30D	23.21	−1133.33	−94.29	−811.48	−5.83	953.09	3.38
	(1.77)	(−3.68)**	(−0.19)	(−1.66)	(−1.51)	(0.82)	(1.80)
$\chi^2 = 156.250^{***}$; degrees of freedom: 152; right−tail p-value 0.3899							
Panel B: 30 industry portfolios							
Constant	−0.02	−0.08	−8.06	−22.49	−0.05	−5.08	0.09
	(−0.18)	(−0.02)	(−1.21)	(−3.86)**	(−0.79)	(−0.37)	(4.96)***
JUNK	−285.17	9559.47	10899.06	6507.00	−288.24	87074.76	−92.48
	(−2.04)	(2.43)	(1.57)	(1.13)	(−5.26)***	(5.41)***	(−3.84)**
DIVY	−16.62	3504.76	−1654.85	614.14	126.85	−19023.10	−5.42
	(−0.24)	(1.73)	(−0.55)	(0.24)	(4.21)**	(−2.95)	(−0.55)
TERM	−1.61	78.49	259.11	47.90	−0.17	370.82	−0.48
	(−0.75)	(1.30)	(2.45)*	(0.57)	(−0.21)	(1.96)	(−2.25)

HB3	11.95	1455.43	−2141.03	−2000.29	20.06	−6105.15	1.09
	(1.24)	(4.42)***	(−3.72)**	(−3.76)**	(6.01)***	(−3.92)***	(0.56)
$\text{DUMMY}_{\text{January}}$	−0.23	−2.04	0.40	22.36	0.02	39.86	−0.02
	(−1.60)	(−0.82)	(0.06)	(5.38)***	(0.83)	(2.77)**	(−1.17)
R_{MKT}	−0.08	12.09	39.60	222.55	−0.12	−11.98	0.07
	(−0.06)	(0.58)	(1.11)	(7.90)***	(−0.34)	(−0.24)	(0.67)
TB30D	−0.57	−3017.96	1024.48	2187.99	−4.82	165.02	1.57
	(−0.02)	(−4.01)**	(0.87)	(2.07)	(−0.58)	(0.06)	(0.44)
$\chi^2 = 138.851$***; degrees of freedom:192; right-tail p-value 0.9985							

This table reports the Generalized Method of moments tests of the moment conditions of the conditional pricing test (Eq. (9) in the text). Panel A reports estimates for the 25 book-to-market and size portfolios; Panel B reports estimates for the 30 industry portfolios. The vectors, δ, ϕ_{MKT}, ϕ_{HML}, ϕ_{SMB}, ϕ_{UMD}, ϕ_{LU}, ϕ_{DISP}, contain the coefficients of the linear relationship between λ, λ_{MKT}, λ_{HML}, λ_{SMB}, λ_{UMD}, λ_{LU}, λ_{DISP} and the vector of instruments, **Z**. (MKT, HML and SMB are the three Fama and French factors, while UMD is the Carhart Momentum factor). Excess rates of return are coded as 1 for a 1% return per month. The factors that proxy for induced uncertainty have been previously orthogonalized by regressing them on the FF factors. The instrumental variables that are expressed as yields or returns are coded as 0.01 for a 1% return per month. The set of instrumental variables include a constant, the one month T-bill yield (TB30D), the dividend yield of the S&P 500 index (DIVY), the term premium-spread between a 10-year and a one-year Treasury bond yield (TERM), the junk premium-spread between Moody's Baa and Aaa corporate bond yields (JUNK), the difference between the one-month returns of a three-month and one-month T-bill (HB3), the excess return of the market (R_{MKT}), and a January dummy that takes value 1 for January and 0 otherwise. All the estimations are done for the period 1964:07–1998:12 (number of observations = 414). We report the estimated coefficient in the first row of each cell. t-statistics is reported below, in parentheses. The last row of each panel reports the χ^2, the number of degrees of freedom and the associated p-value. Asterisks denote the significance level based on Monte-Carlo Bootstrapping. * Indicates significance on 10% level, ** on 5%, and *** on 1% level. Asterisks on χ^2 denote the p-value based on the number of times the χ^2 in the Monte-Carlo replications exceeded the corresponding χ^2 value.

Table 6
Hypothesis testing

Factors in unrestricted model	Null hypothesis	*D* stat.	Degrees of freedom	*p*-value
Panel A: LU and DISP not included among the information variables				
25 book-to-market and size portfolios				
R_{MKT}, LU, DISP	$\phi_{LU}=0,\ \phi_{DISP}=0$	411.75	16	0.000***
	$\phi_{LU}=0$	195.37	8	0.000***
	$\phi_{DISP}=0$	262.92	8	0.000***
R_{MKT}, HML, SMB, LU, DISP	$\phi_{LU}=0,\ \phi_{DISP}=0$	104.84	16	0.000***
	$\phi_{LU}=0$	35.56	8	0.000***
	$\phi_{DISP}=0$	67.57	8	0.000***
R_{MKT}, HML, SMB, UMD, LU, DISP	$\phi_{LU}=0,\ \phi_{DISP}=0$	62.16	16	0.000***
	$\phi_{LU}=0$	27.06	8	0.001***
	$\phi_{DISP}=0$	41.34	8	0.000***
30 industry portfolios				
R_{MKT}, LU, DISP	$\phi_{LU}=0,\ \phi_{DISP}=0$	318.09	16	0.000***
	$\phi_{LU}=0$	135.09	8	0.000***
	$\phi_{DISP}=0$	74.68	8	0.000***
R_{MKT}, HML, SMB, LU, DISP	$\phi_{LU}=0,\ \phi_{DISP}=0$	194.34	16	0.000***
	$\phi_{LU}=0$	75.42	8	0.000***
	$\phi_{DISP}=0$	95.48	8	0.000***
R_{MKT}, HML, SMB, UMD, LU, DISP	$\phi_{LU}=0,\ \phi_{DISP}=0$	121.45	16	0.000***
	$\phi_{LU}=0$	41.74	8	0.000***
	$\phi_{DISP}=0$	62.61	8	0.000***
Panel B: LU and DISP included among the information variables				
25 book-to-market and size portfolios				
R_{MKT}, LU, DISP	$\phi_{LU}=0,\ \phi_{DISP}=0$	633.04	20	0.000***
	$\phi_{LU}=0$	382.02	10	0.000***
	$\phi_{DISP}=0$	299.88	10	0.000***
R_{MKT}, HML, SMB, LU, DISP	$\phi_{LU}=0,\ \phi_{DISP}=0$	714.77	20	0.000***
	$\phi_{LU}=0$	314.95	10	0.000***
	$\phi_{DISP}=0$	136.26	10	0.000***
R_{MKT}, HML, SMB, UMD, LU, DISP	$\phi_{LU}=0,\ \phi_{DISP}=0$	199.01	20	0.000***
	$\phi_{LU}=0$	107.29	10	0.000***
	$\phi_{DISP}=0$	92.57	10	0.000***
30 industry portfolios				
R_{MKT}, LU, DISP	$\phi_{LU}=0,\ \phi_{DISP}=0$	343.09	20	0.000***
	$\phi_{LU}=0$	44.26	10	0.000***
	$\phi_{DISP}=0$	298.41	10	0.000***
R_{MKT}, HML, SMB, LU, DISP	$\phi_{LU}=0,\ \phi_{DISP}=0$	273.45	20	0.000***
	$\phi_{LU}=0$	125.29	20	0.000***
	$\phi_{DISP}=0$	153.02	20	0.000***
R_{MKT}, HML, SMB, UMD, LU, DISP	$\phi_{LU}=0,\ \phi_{DISP}=0$	191.44	20	0.000***
	$\phi_{LU}=0$	83.85	10	0.000***
	$\phi_{DISP}=0$	97.53	10	0.000***

Table 6 (*continued*)

Factors in unrestricted model	Null hypothesis	Degrees of freedom	Periods 1964–1981 *D* stat.	*p*-value	1982–1998 *D* stat.	*p*-value
Panel C: Hypothesis testing in sub-periods						
25 book-to-market and size portfolios						
R_{MKT}, LU, DISP	$\phi_{LU} = 0$, $\phi_{DISP} = 0$	20	1696.16	0.000***	1041.36	0.000***
	$\phi_{LU} = 0$	10	342.16	0.000***	417.12	0.000***
	$\phi_{DISP} = 0$	10	916.16	0.000***	583.16	0.000***
R_{MKT}, HML, SMB, LU, DISP	$\phi_{LU} = 0$, $\phi_{DISP} = 0$	20	450.59	0.000***	628.76	0.000***
	$\phi_{LU} = 0$	10	82.59	0.000***	143.76	0.000***
	$\phi_{DISP} = 0$	10	271.59	0.000***	372.76	0.000***
R_{MKT}, HML, SMB, UMD, LU, DISP	$\phi_{LU} = 0$, $\phi_{DISP} = 0$	20	316.80	0.000***	394.54	0.000***
	$\phi_{LU} = 0$	10	174.42	0.000***	56.66	0.000***
	$\phi_{DISP} = 0$	10	154.04	0.000***	277.94	0.000***
Industry portfolios						
R_{MKT}, LU, DISP	$\phi_{LU} = 0$, $\phi_{DISP} = 0$	20	311.94	0.000***	706.41	0.000***
	$\phi_{LU} = 0$	10	48.59	0.000***	422.14	0.000***
	$\phi_{DISP} = 0$	10	213.53	0.000***	168.67	0.000***
R_{MKT}, HML, SMB, LU, DISP	$\phi_{LU} = 0$, $\phi_{DISP} = 0$	20	684.34	0.000***	218.7	0.000***
	$\phi_{LU} = 0$	10	195.04	0.000***	43.34	0.000***
	$\phi_{DISP} = 0$	10	195.14	0.000***	121.41	0.000***
R_{MKT}, HML, SMB, UMD, LU, DISP	$\phi_{LU} = 0$, $\phi_{DISP} = 0$	20	225.69	0.000***	201.02	0.000***
	$\phi_{LU} = 0$	10	49.12	0.000***	87.45	0.000***
	$\phi_{DISP} = 0$	10	63.16	0.000***	81.41	0.000***

We report the results of the tests of whether induced uncertainty (LU, DISP) is priced. The factors that proxy for induced uncertainty have been previously orthogonalized by regressing them on the FF factors. The first and second column describe the hypothesis that is tested and the alternative. The third, fourth and fifth columns report, respectively, the Newey and West (1987) *D-statistics*, its number of degrees of freedom and the associated *p*-value. This statistics is distributed like as a χ^2. Asterisks denote the significance level based on Monte-Carlo bootstrapping. * Indicates significance on 10% level, ** on 5%, and *** on 1% level. Panel A reports the results for the specification in which the vector of information variables contains a constant, the junk bond premium (JUNK), the difference between the one-month returns of a three-month T-bill and the return of a one-month T-bill (HB3), the term premium (TERM), the dividend yield of the S & P500 index (DIVY), the excess market return (R_{MKT}), the one-month T-bill rate (TB30D), and a January Dummy. Panel B reports the results when the lagged values of LU and DISP are included among the information variables. Panel C reports the results of testing whether LU and DISP are priced in the sub-periods. They are: 1964:07–1981:12 and 1982:01–1998:12.

Carhart momentum factor. [14] We also consider an alternative specification where the restricted model contains just the market portfolio and the unrestricted one contains the market portfolio and the two factors that proxy for induced uncertainty. We also consider specifications where either of the two factors proxying for induced uncertainty is separately considered. We report the values of the estimated coefficients (ϕ_fs) in Table 5, Panels A and B, for the size/book-to-market and industry portfolios, respectively. To assess the incremental explanatory power of the factors proxying for induced uncertainty, we compare the unrestricted specification (inclusive of the factors proxying for induced uncertainty) to the restricted one (not containing these factors). The nice feature of this approach is the fact that the restricted and the unrestricted specifications are nested. We can therefore use the restricted specification as the null hypothesis and the unrestricted one as the alternative and test whether the restrictions of setting the ϕ_fs to zero in Eq. (7) for the additional two factors are rejected. The test is a Newey–West Difference test, based on the difference between the χ^2s of the restricted and unrestricted model (Newey and West, 1987; Eichenbaum et al., 1988). Formally, this corresponds to testing:

$$H_0 : \phi_{\mathrm{LU}} = \phi_{\mathrm{DISP}} = 0 \quad \text{and } H_A : \phi_{\mathrm{LU}} \neq 0 \neq \phi_{\mathrm{DISP}}. \tag{11}$$

We proceed as follows. First, we calculate the χ^2 in the unrestricted model – i.e., six-factor specification. Then, we drop the factors proxying for induced uncertainty, we reestimate the model using a 4-factor specification (i.e., we impose $\phi_{\mathrm{LU}} = \phi_{\mathrm{DISP}} = 0$) *using the same weighting matrix W* as the one estimated in the unrestricted case (six factors) and we calculate the χ^2 in the restricted model. The difference between the two χ^2s is a new χ^2 with the number of degrees of freedom equivalent to the number of new restrictions. This allows us to compare the restricted model with the unrestricted one. If the χ^2 in the restricted model is significantly higher than the χ^2 in the unrestricted one, the null of no pricing of the induced factors is rejected.

The tests of the hypothesis of pricing are reported in Table 6, Panel A, for both the size and book-to-market and industry portfolios. In all the cases, both LU and DISP factors are priced, both jointly and separately. The χ^2 in the restricted model is always significantly higher than the χ^2 in the unrestricted one and the null is always rejected at any confidence level. Also in this case, we address the issue of generated regressors by using the parametric bootstrap approach (Smith and McAleer, 1993).

Up to now, we have implicitly assumed that induced uncertainty does not enter investors' information sets. However, it is possible that investors directly condition on the level of information uncertainty in order to make their portfolio decisions. That is, investors use past measures of information uncertainty in order to derive forecasts of future uncertainty and condition their portfolio decisions on it. In this case, part of the explanatory power of the additional factors could be due simply to the fact that they capture the time variation of investors' information sets. As Ferson and Harvey (1999) pointed out, "simple proxies for time variation in expected returns, based on common instruments are also significant cross-sectional predica-

[14] The factors that proxy for induced uncertainty have been previously orthogonalized by regression on the FF factors.

tors of returns. The ability of these variables to explain the cross-section of returns provides a powerful rejection of the Fama and French model as a conditional asset pricing model." The conditional framework allows us to assess whether induced uncertainty is actually priced or if it only enters as an additional conditioning variable that affects investors' information sets.

We therefore estimate again the conditional specification, including our measures of induced uncertainty among the information variables as well as among the factors. Induced uncertainty is now part of investors' information sets (Ω_{t-1}) as well as part of the factors that affect the price of risk (λs). If the components of induced uncertainty turn out to be significant *even after conditioning on their past values*, this provides direct evidence that these components of induced uncertainty are an additional source of risk against which the market hedges.

We report the tests of the hypothesis of pricing for this specification in Table 6, Panel B, for both the size and book-to-market and industry portfolios. Also in this case, for all the specifications – i.e. one- and four-factor models – the χ^2 in the restricted model is always significantly higher than the χ^2 in the unrestricted one. In all the cases, the factors that proxy for induced uncertainty are priced, both jointly and separately.

These results provide strong support for the theoretical models (Brennan, 1998; Brennan and Xia, 2001b) that argue that investors hedge against information uncertainty. Indeed, not only are informational uncertainty and dispersion of beliefs priced, but they also affect the price of risk. In other words, investors hedge against their fluctuations.

Finally, we test whether the pricing impact of induced uncertainty on prices differs in different periods. We focus on two periods: 1964–1980 and 1981–1998.

The results are reported in Table 6, Panel C. As before, we also apply the parametric bootstrap method to control for the generated regressor problem and report the value of the bootstrapped statistics. The findings are consistent with the earlier results and show that both learning uncertainty and dispersion of beliefs are priced.

As additional robustness check we also considered a specification in which the dispersion of beliefs is proxied by the dispersion of analysts forecasts, constructed by aggregating IBES recommendations. Given the more limited availability of this variable, the sample period is shorter. The results (not reported) are consistent with the previous ones. Dispersion of beliefs is conditionally priced in the case of the industry portfolios as well as in the case of the book-to-market ones. In particular, in the case of the book-to-market portfolios, the sensitivity of the price of the dispersion of beliefs risk to the information variables is significant in the case of the junk premium spread, the dividend yield, the term premium spread, the January dummy and the return of the market portfolio. In the case of the industry portfolios, the sensitivity of the price of the dispersion of beliefs risk to the information variables is significant in the case of the junk premium spread, the dividend yield, the spread between the three-month and one-month T-bill, and the January dummy. Moreover, for all the specifications – i.e., one- and four-factor models – the χ^2 in the restricted model is always significantly higher than the χ^2 in the unrestricted one. That is, the dispersion of beliefs risk is priced. These results suggest that, while the standard factors capture expectations about future fundamentals and innovations about them, our measures of induced uncertainty *capture the dispersion around them*.

6. Conclusion

We empirically investigate "induced" uncertainty and constructed proxies for its two main components: one related to learning uncertainty and one related to dispersion of beliefs. We show that induced uncertainty represents a component of the risk premia that is separate from the fundamental component. We find compelling evidence that both learning uncertainty and dispersion of beliefs are conditionally priced.

These results have implications from a theoretical standpoint as well as from a more empirical one. The identification of the return component related to induced uncertainty may help not only to explain asset returns, but also to deal with unresolved puzzles or apparent market anomalies such as the day-of-the-week effect or momentum. Moreover, it sheds some light on the determinants of trading volume and on its potential explanatory power to predict stock returns. A further promising extension would be to investigate the out-of-sample power of systematic trade in order to predict future asset returns.

Acknowledgements

We thank D. Avramov, A. Basak, B. Biais, P. Boessaerts, O. Bondarenko, B. Dumas, D. Feldman, F. Gomes, R. Jagannathan, P. Maenhout, L. Pastor, P. Sercu, J. Shanken, P. Soderlind, L. Tepla, M. Vassallou, P. Veronesi, two anonymous referees, and the participants of the finance seminar at the Stockholm School of Economics and Summer meeting on Financial Markets (Gerzensee 2002), WFA meeting (Tucson 2002), EFA meeting (Berlin 2002) for their helpful comments and J. Cropper for helpful editorial assistance. A. Simonov thanks Stockholm Institute for Financial Research for support. All the remaining errors are ours.

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