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Conditioning Variables and the Cross Section of Stock Returns

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ABSTRACT

Previous studies identify predetermined variables that predict stock and bond returns through time. This paper shows that loadings on the same variables provide significant cross-sectional explanatory power for stock portfolio returns. The loadings are significant given the three factors advocated by Fama and French (1993) and the four factors of Elton, Gruber, and Blake (1995). The explanatory power of the loadings on lagged variables is robust to various portfolio grouping procedures and other considerations. The results carry implications for risk analysis, performance measurement, cost-of-capital calculations, and other applications.

EMPIRICAL ASSET PRICING is in a state of turmoil. The Capital Asset Pricing Model (CAPM; see Sharpe (1964) and Black (1972)) has long served as the backbone of academic finance and numerous important applications. However, studies have identified empirical deficiencies in the CAPM, challenging its preeminence. The most powerful challenges include market capitalization and related financial ratios that can predict the cross section of returns. For example, the firm "size-effect" drew attention as a challenge to the CAPM. Ratios of stock market price to earnings or the book value of equity are studied by Basu (1977), Banz (1981), Chan, Hamao, and Lakonishok (1991), and Fama and French (1992), among others.

With the CAPM under such strenuous attack the field is hungry for a replacement model.[1] There are some natural heirs waiting in the wings, including the intertemporal equilibrium models of Merton (1973) and Breeden

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[1] The CAPM does have its erstwhile saviors. For example, studies find that dynamic versions of the CAPM with time-varying parameters and/or broader specifications for the market portfolio perform better than traditional formulations of the model. Examples include Harvey (1989), Ferson and Harvey (1991), Pannikkath (1993), Ferson and Korajczyk (1995), Jagannathan and Wang (1996), and Carhart et al. (1996). See Ghysels (1998) for a recent critique of conditional CAPMs.

(1979) and the Arbitrage Pricing Theory of Ross (1976). However, empirical implementations of these models have failed to produce much confidence in their explanatory power (e.g., Chan, Chen, and Hsieh (1985), Chen, Roll, and Ross (1986), Shanken and Weinstein (1990), Hansen and Singleton (1982), Connor and Korajczyk (1988), Lehmann and Modest (1988), and Roll (1995)).

One response to this hunger for a CAPM replacement has been to use the returns of attribute-sorted portfolios of common stocks to represent the factors in a multibeta model. For example, Fama and French (FF) (1993, 1995, 1996) advocate a three-factor "model," in which a market portfolio return is joined by a portfolio long in high book-to-market stocks and short in low book-to-market stocks (HML) and a portfolio that is long in small (i.e, low market capitalization) firms and short in large firms (SMB). Fama and French (1997) use this model for calculating the costs of equity capital for industry portfolios (see also Ibbotson Associates (1998)). Several recent studies use the FF three-factor model as an empirical asset pricing model. However, the model is controversial.

There is controversy over why the firm-specific attributes that are used to form the FF factors should predict returns. Some argue that such variables may be used to find securities that are systematically mispriced by the market (e.g., Graham and Dodd (1934), Lakonishok, Shleifer, and Vishny (1994), Haugen and Baker (1996), and Daniel and Titman (1997)). Others argue that the measures are proxies for exposure to underlying economic risk factors that are rationally priced in the market (e.g., Fama and French (1993, 1995, 1996)). A third view is that the observed predictive relations are largely the result of data snooping and various biases in the data (e.g., Black (1993), MacKinlay (1995), Breen and Korajczyk (1994), Kothari, Shanken, and Sloan (1995); see also Chan, Jegadeesh, and Lakonishok (1995)).

Berk (1995) emphasizes that, because returns are related mechanically to price by a present value relation, ratios that have price in the denominator are related to returns by construction. If the numerator of such a ratio can capture cross-sectional variation in the expected cash flows, the ratio is likely to provide a proxy for the cross section of expected returns. Ratios like the book-to-market are therefore likely to be related to the cross section of stock returns whether they are related to rationally priced economic risks or to mispricing effects. Ferson, Sarkissian, and Simin (1999) illustrate that spread portfolios like SMB or HML can appear to explain the cross section of stock returns even when the attributes used in the sort bear no relationship to risk. Since the FF factors are not derived from a theoretical model, such concerns about their interpretation are natural.

Given the prominence of the Fama–French three-factor model, we believe that it is interesting to test its empirical performance as an asset pricing model. The model was developed to explain unconditional mean (average) returns, and several studies explore its ability to explain average returns.[2]

[2] Fama and French (1993, 1996) find some nonzero alphas relative to the model, but interpret them as economically insignificant. Daniel and Titman (1997) find nonzero alphas using the FF model against a "characteristics-based" alternative for average returns. Berk (1997)

In this paper we test the FF model on conditional expected returns. Thus, we do not focus on alternative "factors" that may provide a better model of average returns. We concentrate instead on the ability of the model to capture common dynamic patterns in returns, modeled using a set of lagged, economy-wide predictor variables. Previous studies, including Fama and French (1996), explore the ability of the FF model to capture dynamic patterns in returns, such as the momentum effect of Jegadeesh and Titman (1993). We focus on common dynamic patterns, captured by a standard set of economy-wide instruments. These lagged instruments are used in numerous previous studies, including some by Fama and French (1988, 1989).

We find that simple proxies for time variation in expected returns, based on common lagged instruments, are also significant cross-sectional predictors of returns. The ability of these variables to explain the cross section of returns provides a powerful rejection of the FF model as a conditional asset pricing model. In some cases loadings on the lagged variables drive out the individual FF variables in cross-sectional regressions. The results are robust to variations in the empirical methods and to a variety of portfolio grouping procedures. We also reject the four-factor model advocated by Elton, Gruber, and Blake (1995). Our results raise a caution flag for researchers who would use the FF and Elton et al. models to control for systematic patterns in risk and expected return. Our results carry implications for risk analysis, performance measurement, cost-of-capital calculations, and other applications.

Our paper is related most closely to studies that use the loadings of stock portfolios on lagged economy-wide variables to explain the cross section of expected returns. Jagannathan and Wang (1996) and Jagannathan, Kubota, and Takehara (1998) show that asset covariances with labor income can be a powerful cross-sectional predictor in the United States and Japan. We use loadings on a larger set of lagged variables from the literature modeling time-series predictability.[3] The results show that size- and book-to-market-related factors leave out important cross-sectional information about expected returns, even in portfolios formed to maximize the potential explanatory power of these variables. The FF factors perform even worse in alternative designs.

The paper is organized as follows. Section I details the empirical methods. Here we propose a simple refinement of the standard Fama–MacBeth (1973) approach to cross-sectional regressions designed to improve its efficiency.

criticizes their sorting procedures and Davis, Fama, and French (1998) question the out-of-sample validity of their findings. Brennan, Chordia, and Subrahmanyam (1998) document cross-sectional attributes such as trading volume and exchange membership which also appear to reject the FF three-factor model.

[3] Conditional asset pricing studies use lagged instruments to model the time series of returns, and then test cross-sectional restrictions on the conditional expected returns. An early example of this approach is the so-called "latent variable" test, pioneered by Hansen and Hodrick (1983) and Gibbons and Ferson (1985); see Ferson, Foerster, and Keim (1993) for a review of this literature. Conversely, a few studies have observed that ratios such as book-to-market, originally identified as a cross-sectional predictor, have some time-series predictive power for aggregate returns (e.g., Pontiff and Schall (1998) and Kothari and Shanken (1997)).

Section II describes the data. Our empirical results are presented in Section III. Section IV explores some of the implications of the results. Section V discusses the robustness of the results to alternative portfolio grouping procedures, errors-in-variables, and other considerations. Some concluding remarks are offered in the final section.

I. The Empirical Framework

A. Time-Series Tests

We start with the null hypothesis that the FF three-factor model identifies the relevant risk in a linear return-generating process:

$$r_{i,t+1} = E_t(r_{i,t+1}) + \beta_{it}'\{r_{p,t+1} - E_t(r_{p,t+1})\} + \epsilon_{i,t+1}, \tag{1}$$

$$E_t(\epsilon_{i,t+1}) = 0,$$

$$E_t(\epsilon_{i,t+1} r_{p,t+1}) = 0,$$

where $r_{i,t+1}$ is the return for any stock or portfolio i, net of the return to a one-month Treasury bill, and $r_{p,t+1}$ is a vector of excess returns on the risk factor-mimicking portfolios. In the FF three-factor model, r_p is a 3×1 vector containing the market index excess return, HML, and SMB. The notation $E_t(\cdot)$ indicates the conditional expectation, given a common public information set at time t. The factor model expresses the unanticipated return, $r_{i,t+1} - E_t(r_{i,t+1})$, as a linear regression on the unanticipated parts of the factors. The third line says that the coefficient vectors β_{it} are the conditional betas of the return r_i on the factors. The error terms $\epsilon_{i,t+1}$ may be correlated across assets.[4]

Equation (1) captures the idea that $r_{p,t+1}$ are risk factors, but it says nothing about the determination of expected returns. We assume the following general model for the conditional expected returns and the betas:

$$E_t(r_{i,t+1}) = \alpha_{it} + \beta_{it}' E_t(r_{p,t+1}),$$

$$\beta_{it} = b_{0i} + b_{1i}' Z_t,$$

$$\alpha_{it} = \alpha_{0i} + \alpha_{1i}' Z_t, \tag{2}$$

where Z_t is an $L \times 1$ vector of mean zero information variables known at time t and the parameters of the model are b_{0i}, b_{1i}, α_{0i}, and α_{1i}. In the FF three-factor model, b_{0i} is 3×1, b_{1i} is $3 \times L$, α_{1i} is $1 \times L$, and α_{0i} is a scalar.

[4] The covariance matrix of these errors would be restricted to have bounded eigenvalues as the number of assets grows in the Arbitrage Pricing Theory, as shown by Chamberlain and Rothschild (1983).

Since we find that the lagged instruments have explanatory power beyond the FF three-factor model, we want to be sure that they do not simply proxy for time-variation in the FF factor betas. Given the evidence of time-varying conditional betas for stock portfolio returns (e.g., Ferson and Harvey (1991), Ferson and Korajczyk (1995), Braun, Nelson, and Sunier (1995)), it makes sense to allow for time-variation in the conditional betas. Thus, we allow the betas in equation (2) to depend on Z_t. The betas are modeled as linear functions of the predetermined instruments, following Shanken (1990), Ferson and Schadt (1996), and other studies. In equation (2), the relation over time between the lagged instruments and the betas for a given portfolio is assumed to be a fixed linear function, as b_{1i} is a fixed coefficient. However, we examine models estimated on rolling sample windows, an approach that allows b_{1i} to vary over time, thus relaxing the assumption of a fixed linear relation.

The hypothesis that the FF model explains expected returns says that the "alpha" term, α_{it}, in equation (2) is zero (i.e., the parameters α_{0i}, α_{1i} are zero). Assuming that alpha is zero is equivalent to assuming that the error term $\epsilon_{i,t+1}$ in equation (1) is not priced. Testing for $\alpha_{1i} = 0$ in system (2) asks whether the variables in Z_t can predict returns over and above their role as linear instruments for the betas.

Equation (2) follows empirical studies in which the alternative hypothesis specifies an alpha that is linear in instrumental variables. Examples include Fama and MacBeth (1973), who use the square of beta and a residual risk; Rosenberg and Marathe (1979), who use firm-specific accounting measures; and Daniel and Titman (1997), who use portfolio valuation ratios. Our example provides a natural test of the FF model, where mispricing related to the lagged, economy-wide instruments Z_t is the alternative hypothesis.

The models for both the betas and the alphas, as given by equation (2), are likely to be imperfect. The second and third equations of (2) may have independent error terms, reflecting possible misspecification of the alphas and the betas.

Combining equations (1) and (2), we derive the following econometric model:

$$r_{it+1} = (\alpha_{0i} + \alpha_{1i}' Z_t) + (b_{0i} + b_{1i}' Z_t) r_{p,t+1} + \epsilon_{i,t+1}. \tag{3}$$

An advantage of regression (3) is that it does not impose a functional form for the expected premiums, $E_t(r_{pt+1})$. This allows us to address the question of whether the lagged market indicators enter as proxies for time-variation in the conditional betas for specific factors, without concern about getting the right model for the expected returns on the factors.

B. Cross-Sectional Test Methodology

The cross-sectional regression approach of Fama and MacBeth (1973) is widely used to study asset pricing models and the cross-sectional structure of asset returns. In this approach returns are regressed each month, cross-sectionally, on a set of predetermined attributes of the firms or portfolios.

The attributes may include estimates of "betas" from a prior time period, as in Fama and MacBeth's study of the CAPM, or they may include other variables such as the book-to-market ratio of the portfolio, as in Fama and French (1992).

A cross-sectional regression using stock returns as the dependent variable is likely to have heteroskedastic and correlated errors, the latter due to the substantial correlation across stock returns in a given month. The usual regression standard errors are therefore not reliable. To test the hypothesis that the expected coefficient is zero, Fama and MacBeth suggest forming a t-ratio as the time series average of the monthly cross-sectional coefficients divided by the standard error of the mean, where the latter is computed from the time-series of the coefficient estimates. Shanken (1992) provides an analysis of the properties of this widely used approach. Jagannathan and Wang (1998) provide a recent asymptotic analysis, and Ahn and Gadarowski (1998) extend the analysis under autocorrelation and heteroskedasticity, where a single cross-sectional regression is used.

In Appendix A of this paper we show that the approach of Fama and MacBeth, which weights the monthly cross-sectional regression coefficients equally over time, can be easily improved. Under standard assumptions, the efficient generalized least squares (GLS) estimator of the pooled time-series and cross-sectional regression can be written as a weighted average of the time series of the Fama–MacBeth coefficients. The monthly estimates are weighted in inverse proportion to their variances. A measure of the total explanatory power of the system is also derived. We present results using the efficient-weighted estimators, as well as using the more traditional approach.

II. The Data

We obtain monthly returns on U.S. common stock portfolios for the period from July 1963 to December 1994. The portfolios are formed similarly to those of Fama and French (1993). Individual common stocks are placed into five groups according to their prior equity market capitalization, and independently on the basis of their ratios of book value to market value per share. This 5×5 classification scheme results in a sample of 25 equity portfolio returns. The appendix provides a more detailed description and Table I presents summary statistics for the returns. The means and standard deviations are annualized.

Our lagged instrumental variables, Z_t, follow from previous studies. These are: (1) the difference between the one-month lagged returns of a three-month and a one-month Treasury bill ("hb3"; see Campbell (1987), Harvey (1989), Ferson and Harvey (1991)); (2) the dividend yield of the Standard and Poors 500 (S&P 500) index ("div"; see Fama and French (1988)); (3) the spread between Moody's Baa and Aaa corporate bond yields ("junk"; see Keim and Stambaugh (1986) or Fama (1990)); (4) the spread between a ten-year and a one-year Treasury bond yield ("term"; see Fama and French (1989));

Table I

Summary Statistics

Returns on 25 value-weighted portfolios formed on size (as of June of the preceding year) and the ratio of book value to market value (as of the previous December) are summarized. Returns are measured in excess of a one-month Treasury bill return. S1 refers to the lowest 20 percent of market capitalization, S5 is the largest 20 percent, B1 refers to the lowest 20 percent of the book/market ratios, and B5 is the largest 20 percent. Market is the return on the value-weighted portfolio of all COMPUSTAT stocks used in forming the portfolios. HML is a high book/market less a low book/market return and SMB is a small firm return less a large firm return, as described in the text. The sample period is July 1963 through December 1994, which provides 378 observations. The sample means are annualized by multiplying by 12 and the sample standard deviations are multiplied by $12^{1/2}$. ρ_j is the sample autocorrelation at lag j.

Portfolio	Mean	Std. Dev.	ρ_1	ρ_2	ρ_3	ρ_4	ρ_{12}	ρ_{24}
S1/B1	8.89	26.18	0.21	0.02	−0.01	0.01	0.09	−0.01
S1/B2	14.18	23.01	0.20	0.00	−0.01	−0.00	0.10	−0.02
S1/B3	15.41	20.93	0.23	−0.01	−0.01	−0.02	0.14	−0.00
S1/B4	17.20	19.90	0.21	−0.01	−0.01	−0.02	0.16	−0.01
S1/B5	18.68	20.92	0.23	−0.02	−0.03	−0.04	0.22	0.06
S2/B1	11.60	24.35	0.16	−0.02	−0.02	−0.03	0.02	−0.06
S2/B2	14.36	21.34	0.17	−0.03	−0.02	−0.02	0.08	0.02
S2/B3	16.53	19.47	0.16	−0.04	−0.04	−0.02	0.09	−0.04
S2/B4	16.81	17.86	0.15	−0.04	−0.03	−0.01	0.12	0.01
S2/B5	18.55	20.34	0.16	−0.07	−0.07	−0.04	0.15	0.03
S3/B1	11.12	22.27	0.15	−0.02	−0.03	−0.05	0.02	−0.04
S3/B2	13.80	18.86	0.16	−0.03	−0.00	−0.04	0.05	−0.01
S3/B3	14.61	17.44	0.14	−0.02	−0.04	−0.03	0.03	−0.01
S3/B4	16.11	16.35	0.13	−0.04	−0.02	−0.04	0.09	0.06
S3/B5	18.48	18.78	0.11	−0.10	−0.06	−0.03	0.10	0.00
S4/B1	11.89	20.03	0.11	−0.02	−0.02	−0.02	0.01	−0.03
S4/B2	10.59	18.00	0.10	−0.04	−0.02	−0.02	0.01	−0.00
S4/B3	13.36	17.01	0.07	−0.05	−0.02	−0.06	0.02	0.00
S4/B4	15.21	16.44	0.07	−0.03	−0.03	−0.05	0.08	0.01
S4/B5	18.01	19.36	0.06	−0.04	−0.02	−0.02	0.06	−0.00
S5/B1	10.45	16.52	0.05	−0.01	−0.02	−0.01	0.05	−0.01
S5/B2	10.49	15.78	0.03	−0.06	0.00	−0.00	−0.00	−0.02
S5/B3	10.39	14.65	−0.05	−0.07	0.01	0.01	0.00	0.02
S5/B4	12.40	14.35	−0.07	0.01	0.05	−0.08	0.04	0.01
S5/B5	14.40	16.78	−0.02	−0.00	−0.03	−0.03	0.06	0.01
Market	11.26	15.12	0.04	−0.04	−0.01	−0.01	0.03	−0.01
SMB	3.23	9.91	0.18	0.06	−0.02	0.04	0.22	0.05
HML	5.40	8.88	0.20	0.06	−0.01	−0.06	0.10	0.10

and (5), the lagged value of a one-month Treasury bill yield ("Tbill"; see Fama and Schwert (1977), Ferson (1989), or Breen, Glosten, and Jagannathan (1989)).[5]

Table II summarizes time-series regressions of the 25 portfolios on the lagged instruments. The data are monthly for the July 1963 to December 1994 period. The regressions produce significant t-statistics for many of the variables. The adjusted R-squares vary from about six to 14 percent across

[5] Because of concerns about possible nonstationarity of the bill, we also examine results where the one-month yield is stochastically detrended by subtracting the lagged, twelve-month moving average.

Table II

In-Sample Predictability of Size and Book/Market Portfolios

Monthly excess returns are regressed on a set of lagged instrumental variables. The instrumental variables include "hb3," the lagged difference between three-month and one-month T-bill returns; "div," the lagged S&P 500 dividend yield; "junk," the lagged spread between Moody's Baa and Aaa yields; "term," the lagged spread between the 10-year and three-month Treasury yields. "Tbill" is the yield on the Treasury bill closest to 30 days to maturity from CRSP. The sample is July 1963 to December 1994 and the number of observations is 378. Returns on 25 value-weighted portfolios formed on size and the ratio of book value to market value are measured in excess of the return on a 30-day Treasury bill. S1 refers to the lowest 20 percent of market capitalization, S5 is the largest 20 percent, B1 refers to the lowest 20 percent of the book/market ratios and B5 is the highest 20 percent. Market is the return on the value-weighted portfolio of all COMPUSTAT stocks used in excess of the Ibbotson 30-day bill rate. HML is a high book/market less a low book/market return and SMB is a small firm return less a large firm return, as described in the text. Heteroskedasticity consistent t-ratios are on the second line below the coefficients. "R^2" is the coefficient of determination of the regression, with the adjusted R-square shown on the second line. "Autocorr" is the first-order autocorrelation of the regression residual, with its t-statistic on the second line.

	Variables							
	constant	hb3	div	junk	term	Tbill	R^2	Autocorr
S1/B1	−3.75	3.81	3.57	3.53	−1.24	−23.23	0.15	0.13
	−2.07	0.95	5.01	2.92	−3.40	−6.17	0.14	2.36
S1/B2	−3.21	2.93	2.83	3.33	−0.81	−18.27	0.14	0.12
	−2.00	0.89	4.26	3.11	−2.40	−5.06	0.13	2.20
S1/B3	−2.39	3.85	2.54	3.35	−0.83	−17.56	0.15	0.15
	−1.71	1.30	4.34	3.36	−2.81	−5.49	0.14	2.57
S1/B4	−1.50	4.63	2.24	3.13	−0.80	−16.51	0.15	0.13
	−1.13	1.73	3.99	3.31	−2.77	−5.30	0.14	2.26
S1/B5	−1.57	4.80	2.41	3.20	−0.83	−17.47	0.15	0.15
	−1.14	1.73	3.94	3.11	−2.73	−5.25	0.14	2.52
S2/B1	−3.05	4.11	2.98	2.86	−0.85	−19.19	0.13	0.09
	−1.74	1.06	4.22	2.47	−2.48	−5.36	0.12	1.73
S2/B2	−3.22	4.49	2.71	2.99	−0.76	−16.87	0.15	0.08
	−2.19	1.54	4.35	3.07	−2.51	−5.17	0.14	1.62
S2/B3	−1.70	5.68	1.95	2.70	−0.58	−13.69	0.13	0.08
	−1.26	2.09	3.36	2.94	−2.05	−4.39	0.12	1.46
S2/B4	−2.42	5.77	2.04	2.39	−0.51	−12.47	0.15	0.05
	−2.08	2.29	4.03	2.74	−1.91	−4.33	0.14	0.86
S2/B5	−1.67	6.56	2.15	2.46	−0.68	−14.35	0.13	0.09
	−1.24	2.35	3.57	2.50	−2.32	−4.41	0.12	1.56
S3/B1	−2.88	4.62	2.51	2.84	−0.70	−16.40	0.13	0.08
	−1.82	1.33	3.89	2.71	−2.17	−4.86	0.12	1.44
S3/B2	−2.50	5.89	2.15	2.79	−0.60	−14.23	0.16	0.07
	−1.94	2.20	3.86	3.14	−2.15	−4.79	0.15	1.30
S3/B3	−2.21	4.75	1.91	2.37	−0.45	−12.22	0.14	0.03
	−1.85	1.89	3.82	2.85	−1.79	−4.51	0.13	0.62
S3/B4	−0.63	5.64	1.61	2.10	−0.58	−12.10	0.13	0.03
	−0.57	2.48	3.48	2.69	−2.38	−4.64	0.12	0.56
S3/B5	−1.57	6.39	1.94	1.76	−0.55	−11.75	0.11	0.05
	−1.21	2.34	3.44	2.00	−2.01	−3.86	0.10	0.92

Table II—*Continued*

	Variables							
	constant	hb3	div	junk	term	Tbill	R^2	Autocorr
S4/B1	−1.99	6.60	2.00	2.27	−0.61	−13.41	0.12	0.04
	−1.40	1.97	3.48	2.31	−2.05	−4.56	0.11	0.84
S4/B2	−2.67	5.33	1.97	2.29	−0.46	−12.30	0.14	0.01
	−2.07	1.91	3.66	2.62	−1.70	−4.33	0.13	0.13
S4/B3	−1.67	4.97	1.69	2.56	−0.55	−12.04	0.14	0.04
	−1.47	2.18	3.39	3.20	−2.17	−4.38	0.12	0.67
S4/B4	−0.66	5.08	1.38	2.07	−0.48	−10.54	0.11	0.02
	−0.58	2.06	2.97	2.52	−1.97	−4.04	0.10	0.46
S4/B5	−0.90	6.39	1.59	2.55	−0.59	−12.11	0.11	0.03
	−0.67	2.36	2.83	2.74	−2.17	−4.02	0.10	0.48
S5/B1	−0.67	4.90	1.00	1.85	−0.30	−8.33	0.08	0.00
	−0.58	1.54	1.93	2.14	−1.24	−3.32	0.07	0.03
S5/B2	−1.87	4.51	1.31	1.85	−0.26	−8.32	0.10	0.04
	−1.70	1.65	2.76	2.29	−1.08	−3.37	0.09	0.63
S5/B3	−1.74	4.46	1.24	1.06	−0.12	−6.66	0.09	−0.11
	−1.72	1.66	2.81	1.37	−0.53	−2.91	0.07	−1.71
S5/B4	−0.86	3.19	1.02	1.89	−0.34	−7.57	0.08	−0.16
	−0.87	1.34	2.58	2.61	−1.58	−3.32	0.07	−2.89
S5/B5	0.14	5.34	0.78	2.32	−0.41	−8.46	0.08	−0.09
	0.12	2.13	1.64	2.94	−1.61	−3.19	0.07	−1.66
Market	−1.49	5.27	1.36	1.82	−0.33	−9.16	0.12	−0.04
	−1.43	1.95	2.93	2.40	−1.47	−3.91	0.11	−0.71
SMB	−1.20	−0.02	1.22	0.93	−0.40	−7.36	0.10	0.11
	−1.81	−0.01	4.68	1.97	−2.97	−5.36	0.08	1.92
HML	1.07	0.45	−0.46	−0.25	0.08	2.50	0.02	0.20
	1.77	0.28	−1.84	−0.48	0.52	1.70	0.00	3.10

the 25 portfolios. The residual autocorrelations are generally not large—approximately 0.1 on average—but there are some statistically significant autocorrelations for the small-firm portfolios. These no doubt reflect the nonsynchronous trading of these small stocks.[6]

The coefficients on the lagged variables show a great deal of spread across the portfolios. This is important, as cross-sectional dispersion in the coefficients is necessary to provide explanatory power for the cross section of stock returns.

Table II also reports regressions for the FF factor portfolios on the lagged instruments. Two of the FF factors, MARKET and SMB, produce similar R-squares to the 25 portfolios, but the HML portfolio is remarkable because its adjusted R-square is zero. This foreshadows the result that the HML portfolio does not help to explain time-varying conditional expected returns.

[6] The autocorrelations are estimated by regressing the fitted residual on its lagged value by OLS. A White (1980) t-ratio is reported for the slope coefficient of this regression in Table II.

III. Empirical Evidence

A. Are the Betas Time-Varying?

As we show later, the lagged instruments track variation in expected returns that is not captured by the FF three-factor model. However, the lagged instruments may have explanatory power because they pick up time variation in the betas on the FF factors. This would imply that the FF model should be implemented in a conditional form—that is, with time-varying betas—but it would not indicate a fundamental shortcoming of the FF model.[7]

To examine the issue of time-varying betas, we report regressions in which we allow the lagged instruments to enter the models through the conditional betas. Table III presents the results of estimating the time-series regression (3) for each of the 25 portfolio returns. Both one-factor models, where the CRSP index is the market factor, and the FF three-factor model are examined; to save space we focus on the three-factor model in Table III.[8] The table reports the adjusted R-squares of the regressions and the right-tailed p-values of F-tests for the hypothesis that the interaction terms between the factor-mimicking portfolios and the lagged variables may be excluded from the regressions. In the three-factor model, the F-tests for 11 of the 25 portfolios produce p-values below 0.05 when the alphas are allowed to be time varying, and 12 cases reject constant betas on the assumption that the alphas are constant over time. A joint Bonferroni test strongly rejects the hypothesis that the betas are constant over time, in either specification. The evidence of Table III suggests that even if the FF factors are useful to control for "risk," it may be important to allow for the time-varying betas picked up by the lagged instruments.

B. Time-Series Evidence on the Three-Factor Model

Table IV presents further results from the time-series model given in equation (3). For the first two columns we regress the 25 size and book/market portfolio excess returns on a constant and the three FF factors. A t-test is conducted for the hypothesis that the intercept is equal to zero, similar to the results of Fama and French (1993, 1996), who found that the intercepts were close to zero. The null hypothesis is equivalent to the statement that a constant combination of the three FF factors is an unconditional (fixed-weight) minimum variance portfolio. This says that the three factors explain the unconditional expected returns of the 25 portfolios and, therefore, all fixed-weight portfolios formed from them. Like Fama and French (1993, 1996), we find little evidence against this hypothesis. Only four of 25 p-values

[7] Subsequent to an earlier version of this paper, Fama and French (1997) presented evidence of time-varying betas in their model when applied to industry portfolios. Eckbo, Norli, and Masulis (1998) provide evidence of time-varying betas for firms issuing new equity and their matching firms.

[8] More details are available at http://www.duke.edu/~charvey/Research/index.htm.

Table III

Tests for Time-Varying Betas in a Three-Factor Model

Returns on 25 value-weighted portfolios are measured in excess of the return on a 30-day Treasury bill and regressed on lagged instrumental variables, the excess returns of three-factor portfolios, the three-factor returns each multiplied by the instrumental variables, and a constant. The adjusted R-square of this regression is shown in the second column. A restricted regression is estimated where the portfolio returns are regressed only on the three-factor portfolios, the lagged instruments, and a constant. The p-value of an F-test comparing the two R-squares is presented in the third column, as a test for time-varying betas. In the three right-most columns a similar experiment is conducted (constant alphas), in which the lagged instruments do not appear except as interaction terms. The three factor-portfolios are the market return, a small minus large firm portfolio, and a high minus low book-to-market portfolio. The lagged instrumental variables are described in Table II. The sample period is July 1963 through December of 1994 and the number of observations is 378. S1 refers to the lowest 20 percent of market capitalization, S5 is the largest 20 percent, B1 refers to the lowest 20 percent of the book/market ratios, and B5 is the highest 20 percent. Bonferroni is the upper bound on the p-value of a joint test across the portfolios. #<0.05 is the number of p-values less than 0.05.

	Time-Varying Alphas			Constant Alphas		
Portfolio	R^2 Constant Betas	R^2 Time-Varying Betas	F-test (p-value)	R^2 Constant Betas	R^2 Time-Varying Betas	F-test (p-value)
S1/B1	0.673	0.685	0.002	0.651	0.659	0.014
S1/B2	0.693	0.703	0.004	0.681	0.689	0.020
S1/B3	0.688	0.701	0.001	0.673	0.682	0.012
S1/B4	0.647	0.663	0.001	0.633	0.645	0.007
S1/B5	0.608	0.624	0.002	0.592	0.604	0.008
S2/B1	0.783	0.787	0.037	0.774	0.777	0.125
S2/B2	0.786	0.795	0.002	0.775	0.779	0.047
S2/B3	0.758	0.769	0.001	0.756	0.763	0.009
S2/B4	0.765	0.775	0.001	0.758	0.764	0.019
S2/B5	0.706	0.721	0.000	0.702	0.711	0.007
S3/B1	0.835	0.838	0.040	0.832	0.834	0.107
S3/B2	0.845	0.850	0.006	0.838	0.839	0.210
S3/B3	0.803	0.807	0.026	0.800	0.801	0.147
S3/B4	0.795	0.800	0.018	0.791	0.794	0.057
S3/B5	0.730	0.737	0.013	0.729	0.733	0.069
S4/B1	0.879	0.878	0.730	0.878	0.877	0.736
S4/B2	0.900	0.904	0.001	0.898	0.901	0.004
S4/B3	0.861	0.862	0.126	0.859	0.859	0.272
S4/B4	0.785	0.787	0.199	0.786	0.788	0.152
S4/B5	0.751	0.761	0.002	0.751	0.761	0.003
S5/B1	0.878	0.881	0.024	0.877	0.878	0.179
S5/B2	0.911	0.913	0.010	0.911	0.914	0.008
S5/B3	0.832	0.837	0.009	0.831	0.836	0.012
S5/B4	0.772	0.773	0.356	0.774	0.775	0.231
S5/B5	0.643	0.648	0.067	0.644	0.649	0.062
Bonferroni			0.001			0.001
# < 0.05			11			12

Table IV

Time-Varying Alphas in a Three-Factor Model

The first column shows the average annualized intercept (monthly figure × 12, in percent) in a regression of the portfolio excess return on a constant and three-factor portfolios. The second column presents the right-tailed p-value of a heteroskedasticity consistent test of whether this intercept is equal to zero. The third column reports the p-value of an F-test of whether the intercept is constant in a model with constant betas. The fourth column reports p-values of an F-test of the hypothesis that the intercept is constant in the model with time-varying betas. The alternative for the constant alpha tests is to model the alphas as linear functions of the lagged instrumental variables. The three-factor portfolios are the market return, a small minus large firm portfolio, and a high minus low book/market portfolio. The lagged instrumental variables are described in Table II. The sample is July 1963 to December 1994 and the number of observations is 378. Returns on 25 value-weighted portfolios formed on size and the ratio of book value to market value are measured in excess of the return on a 30-day Treasury bill. S1 refers to the lowest 20 percent of market capitalization, S5 is the largest 20 percent, B1 refers to the lowest 20 percent of the book/market ratios, and B5 is the highest 20 percent. Bonferroni is the upper bound on the p-value of a joint test across the portfolios. #<0.05 is the number of p-values less than 0.05.

Portfolio	Annual Intercept (Constant alpha, constant betas)	Test Zero Unconditional Alpha	Test Constant Alpha (Constant betas)	Test Constant Alpha (Time-varying betas)
S1/B1	−6.036	0.000	0.000	0.000
S1/B2	−1.924	0.036	0.002	0.002
S1/B3	−0.880	0.237	0.000	0.000
S1/B4	0.585	0.425	0.000	0.000
S1/B5	0.170	0.815	0.000	0.000
S2/B1	−0.917	0.320	0.002	0.002
S2/B2	−0.465	0.551	0.000	0.001
S2/B3	0.893	0.274	0.001	0.001
S2/B4	0.723	0.303	0.042	0.050
S2/B5	0.034	0.966	0.000	0.000
S3/B1	−1.100	0.239	0.000	0.000
S3/B2	0.100	0.908	0.002	0.003
S3/B3	−0.347	0.683	0.002	0.003
S3/B4	0.672	0.408	0.003	0.004
S3/B5	0.960	0.294	0.001	0.001
S4/B1	1.324	0.154	0.004	0.005
S4/B2	−2.322	0.011	0.000	0.000
S4/B3	−0.963	0.310	0.000	0.000
S4/B4	−0.040	0.969	0.000	0.000
S4/B5	0.476	0.693	0.015	0.019
S5/B1	2.295	0.002	0.000	0.000
S5/B2	−0.683	0.413	0.000	0.000
S5/B3	−1.240	0.222	0.000	0.000
S5/B4	−1.457	0.102	0.002	0.003
S5/B5	−1.678	0.196	0.079	0.092
Bonferroni	—	0.000	0.000	0.000
# < 0.05	—	4	24	24

(second column) are less than 0.05. The largest unconditional alpha is for the small-firm/value portfolio and is just over six percent per year; the second largest alpha is about 2.3 percent per year.

In the third column of Table IV we subject the FF model to a more stringent test, with a specific alternative hypothesis. We regress the portfolio excess returns over time on the three FF factors and the vector of lagged instruments. The F-test for the hypothesis that the lagged variables may be excluded from the regression is reported. This is implied by the hypothesis that the FF three-factor model with constant betas can explain the dynamic behavior of the *conditional* expected returns of the 25 portfolios, given the lagged instruments. Now we find strong evidence against the model. All of the p-values are less than 0.10, and all except one of the 25 are less than 0.05.[9]

Since we find evidence that conditional betas for the 25 portfolios on the FF variables are time-varying, the instruments could enter the model through the betas. In other words, by holding the betas fixed, the tests may be biased against the FF model. In the fourth column of Table IV we allow the betas to be time-varying. Each portfolio excess return is regressed on a constant intercept, the lagged instruments, the FF factors, and the products of the FF factors with the lagged instruments. This allows the FF factor betas to vary as a linear function of the lagged instruments. The null hypothesis that the alphas are constant (the lagged instruments may be excluded from the model of alpha) is tested with an F-test. Most of the p-values from this test are again small. We thus obtain a strong rejection of the FF three-factor model, even allowing for time-varying betas that depend on the instruments.

In summary, Fama and French (1993) found that the regression intercepts are close to zero for their three-factor model. However, conditional on the lagged instruments the alphas are time-varying and thus not zero. This implies that the FF three-factor model does not explain the conditional expected returns of these portfolios. Even a conditional version of the FF model, with time-varying betas, can be rejected.

C. Economic Significance of the Conditional Alphas

Though the time-series tests reject the FF model, the lagged instruments deliver only small increments to the already large time-series R-squares provided by the contemporaneous factors. We therefore conduct experiments to assess the economic significance of the conditional alphas.

In a first experiment we use the conditional alphas in a step-ahead "trading strategy" to assess the economic significance of the departures from the FF model. Each month we form portfolios using the conditional alphas of equation (3) estimated with trailing data. Each of the 25 size-sorted and book-to-market-sorted portfolios is assigned an alpha rank, and an equally

[9] Conditional pricing implies that the intercepts and the slopes on the lagged instruments are zero; we test the weaker implication that only the slopes are zero. Including the intercept would provide an even more powerful rejection of the FF model.

weighted combination of the top seven and bottom seven alpha portfolios is formed and held for one month. The procedure is repeated each month, producing a time-series of trading strategy returns for high-alpha and low-alpha portfolios. The models are estimated using either an expanding sample or a rolling, 60-month sample. We find that the subsequent returns of the high conditional alpha portfolios exceed those of the low conditional alpha portfolios by economically significant amounts. With the expanding sample, the difference in return is more than nine percent per year. With the rolling sample, it is more than eight percent per year. The standard deviations of the returns are slightly smaller in the high-alpha portfolios, which reinforces the economic significance of the conditional alphas.

In a second experiment we use the fitted values of the alphas, $\alpha_{0i} + \alpha'_{1i} Z_t$, from equation (3) in monthly cross-sectional regressions for $r_{i,t+1}$, where equation (3) is estimated using trailing data only. The three-factor betas for time t are also included in the regression. This means that the cross-sectional regression coefficient on the fitted alphas is the return for the month to a zero-net investment portfolio with three-factor betas equal to zero and a fitted alpha, based on past data, of one percent per month. If the FF model is correctly specified, the expected return of such a portfolio, and therefore the expected time-series average of the coefficient, should be zero.

The results of the cross-sectional regressions using a number of specifications for the fitted alphas and the FF factor betas may be found on the Internet. The results show that the fitted alphas are significant regressors in models with the three FF betas, producing t-ratios between 4.3 and 7.8, depending on the experiment. Including the fitted alphas in the regressions does not have much effect on the coefficients on the FF betas because the fitted alphas are constructed to be orthogonal to the FF betas in the cross section.[10] Thus, the regressions further illustrate the economic significance of the conditional alphas.

D. The Cross Section of Expected Stock Returns Revisited

Fama and French (1992) use cross-sectional regressions of stock portfolio returns on size and book-to-market to attack the CAPM. In this section we use a similar approach to examine the FF three-factor model in more detail. Consider the cross-sectional regression

$$r_{it+1} = \gamma_{o,t+1} + \gamma'_{t+1} \beta_{it} + \gamma_{4,t+1} \delta'_{it} Z_t + e_{it+1}; \qquad i = 1,\ldots,N, \tag{5}$$

[10] This occurs because the factors are simple combinations of the test assets, which implies that a weighted average of the alphas must be zero. Consider the special case of a stacked regression model: $r = \alpha + r_p \beta + u$, where $r_p = rW$ is a combination of the test assets with weight given by the $n \times k$ matrix, W. Using the definition $\beta = (W'VW)^{-1} W'V$, where V is the covariance matrix of r, it is easy to show that $\alpha' V^{-1} \beta' = 0$.

where $\gamma_{o,t+1}$ is the intercept and $\gamma_{t+1} = (\gamma_{1,t+1}, \gamma_{2,t+1}, \gamma_{3,t+1})'$ and $\gamma_{4,t+1}$ are the slope coefficients. The β_{it} are the betas on the three FF variables, formed using information up to time t. The term $\delta_{it}' Z_t$ denotes the fitted conditional expected return, formed by regressing the return i on the lagged variables Z, using data up to date t, where δ_{it} is the time-series regression coefficient.[11] We use fit_{it} as a shorthand for this variable. The dating convention thus indicates when a coefficient or variable would be public information. The hypothesis that the FF factor betas explain the cross section of expected returns implies that the coefficient $\gamma_{4,t+1}$ is zero. The alternative hypothesis is that the FF variables do not explain the conditional expected returns, as captured by the lagged instruments.

Jagannathan and Wang (1998) study the asymptotic properties of cross-sectional regression models, allowing for heteroskedasticity in returns. They show that if an asset pricing model is misspecified, the coefficients are biased and, in some cases, the t-ratios do not conform to a limiting t distribution. Thus, the coefficients cannot be used to select significant factors. They emphasize, however, that including additional cross-sectional predictors in the model, the t-ratios for those variables provide a valid test of the null model. Their analysis justifies our use of the t-ratio on γ_4 as a test of the FF three-factor model.

Table V summarizes several versions of the cross-sectional regressions. The time-series averages of the cross-sectional regression coefficients are shown along with their Fama–MacBeth t-ratios. We examine one-factor models, where the CRSP value-weighted index is the factor, and three-factor models using the FF variables. Table V concentrates on the FF three-factor model.[12] We estimate the betas using either an expanding sample or a rolling, 60-month prior estimation period. When conditional betas are used (Panels C, D, G, and H) they are assumed to be linear functions of the lagged instruments. We estimate each cross-sectional regression model with and without the fitted expected returns in the regression, and we compare the results.

The FF model implies that the intercepts of the cross-sectional regressions should be zero. Table V shows that when the three-factor betas are the only regressors the intercept has a t-ratio of 0.80 using the expanding sample, and as large as 1.9 in other cases. The larger values may be interpreted as weak evidence against the FF three-factor model, similarly to Fama and French (1993, 1996).

When the fitted expected returns using the lagged market instruments (the "*fit*") are included in the cross-sectional regressions the results are dramatically different. The t-ratios of the *fit* are in excess of 5.7 in all of the

[11] The time-series regression is $r_{i\tau} = \delta_{it}' Z_{\tau-1} + v_{i\tau}$, $\tau = 1, \ldots, t$, so δ_{it} is estimated using data up to time t for returns and up to time $t-1$ for the lagged instruments.

[12] Results for the one-factor models are available on the Internet. Consistent with Fama and French (1992), there is no significant relation between the returns on these portfolios and the market index betas. However, the fitted expected returns using the lagged market instruments are highly significant, with t-ratios in excess of seven.

Table V

Evidence on the Cross Section of Stock Returns

The average coefficients from monthly cross-sectional regressions are expressed as percentage per month. The dependent variables are value-weighted portfolio returns at time t, formed on size and the ratio of book-to-market, measured in excess of the return on a 30-day Treasury bill. The regressors are a constant, the betas on the three FF factors, and a fitted conditional expected return estimated with data up to time $t-1$. The betas are from a time-series regression of the portfolio excess returns on the excess factor returns. The three FF factors are the market return (mkt), a small minus large market capitalization portfolio (smb), and a high minus a low book-to-market portfolio (hml). The fitted expected return is from a time-series regression of the portfolio return on lagged instrumental variables, using data to time $t-1$. The instrumental variables used to form the fitted expected return (*fit)* are described in Table II. The sample is July 1963 to December 1994 and the number of time-series observations is 378. The number of cross-sectional regressions is 377. For the first 60 months we use the in-sample betas. After observation 60, the sample for estimating the beta grows by one observation in Panel A. In Panel B, the regressions use a 60-month rolling window to estimate the betas (the time-series predicted returns use an expanding sample). t-statistics are reported under the average coefficients. γ_0 is the average intercept.

γ_0	γ_1(mkt)	γ_2(smb)	γ_3(hml)	γ_4(fit)
Panel A. With Expanding Sample Betas				
0.230	0.190	0.198	0.495	—
0.804	0.586	1.354	3.648	—
0.502	—	—	—	0.510
2.036	—	—	—	6.030
0.041	0.322	0.073	0.232	0.466
0.137	0.953	0.496	1.588	7.797
Panel B. With 60-Period Rolling Sample Betas				
0.483	−0.049	0.208	0.473	—
1.865	−0.167	1.426	3.563	—
0.502	—	—	—	0.510
2.036	—	—	—	6.030
0.227	0.153	0.092	0.237	0.445
0.803	0.491	0.631	1.715	7.537
Panel C. With Expanding Sample Conditional Betas				
0.217	0.235	0.195	0.416	—
0.872	0.974	1.426	3.473	—
0.502	—	—	—	0.510
2.036	—	—	—	6.030
0.201	0.341	0.173	0.176	0.387
0.785	1.392	1.284	1.411	6.659
Panel D. With 60-Period Rolling Sample Conditional Betas				
0.190	0.276	0.195	0.360	—
0.868	1.548	1.508	3.392	—
0.502	—	—	—	0.510
2.036	—	—	—	6.030
0.254	0.211	0.160	0.205	0.355
1.138	1.243	1.293	2.041	6.250

Table V—*Continued*

γ_0	γ_1(mkt)	γ_2(smb)	γ_3(hml)	γ_4(fit)
Panel E. WLS with Expanding Sample Betas				
0.236	0.186	0.229	0.466	—
0.826	0.555	1.561	3.482	—
0.467	—	—	—	0.523
1.859	—	—	—	6.107
0.066	0.301	0.106	0.246	0.435
0.219	0.880	0.718	1.714	7.438
Panel F. WLS with 60-Period Rolling Sample Betas				
0.508	−0.070	0.242	0.440	—
1.929	−0.232	1.660	3.297	—
0.497	—	—	—	0.505
1.986	—	—	—	5.834
0.350	0.036	0.123	0.250	0.391
1.251	0.117	0.852	1.815	6.943
Panel G. WLS with Expanding Sample Conditional Betas				
0.227	0.216	0.236	0.403	—
0.923	0.870	1.721	3.413	—
0.463	—	—	—	0.527
1.844	—	—	—	6.164
0.212	0.319	0.202	0.204	0.349
0.830	1.278	1.497	1.673	6.307
Panel H. WLS with 60-Period Rolling Sample Conditional Betas				
0.208	0.246	0.247	0.345	—
0.960	1.360	1.893	3.220	—
0.502	—	—	—	0.511
1.992	—	—	—	5.866
0.314	0.158	0.196	0.211	0.324
1.430	0.924	1.587	2.105	5.730

panels. The FF three-factor model thus fails miserably when confronted with this alternative hypothesis. Although the magnitudes of the coefficients are difficult to interpret if the model is misspecified (Jagannathan and Wang (1998)), some of the patterns are interesting. With the *fit* in the regressions the coefficients on HML are consistently smaller, and the t-ratios become individually insignificant in many of the cases. The average coefficient on the market beta, $\gamma_1(mkt)$, is usually larger in the presence of the fit. The intercepts are typically smaller and insignificant when the fit is included.

The coefficients and t-ratios in Table V show that the FF three-factor model is rejected. The *fit* thus provides a powerful alternative that allows us to detect patterns in the cross section of the conditional expected returns that the FF model does not capture. The rejection can also be turned around. If the fit delivered a perfect proxy for $E_t(r_{i,t+1})$, then in the cross section, the coefficients on β_{it} should have a mean of zero and the coefficient on the *fit* should be 1.0. The tests therefore reject the hypothesis that the *fit* completely captures expected returns. Of course, since the lagged instruments represent only a subset of publicly available information, and the regressions that determine the *fit* have estimation error, we do not expect the *fit* to provide a perfect proxy for expected returns. We discuss errors-in-variables in Section 5.A below.

The t-ratios in Table V allow a convenient economic interpretation of the rejections as they are proportional to a portfolio's Sharpe ratio (average excess return divided by standard deviation). For example, with a sample of 378 months and a t-ratio for the HML premium of 3.65 in Panel A, the Sharpe ratio of the HML premium is $3.65/\sqrt{378} = 0.188$. MacKinlay (1995) argues that such a high Sharpe ratio for monthly stock returns is implausible. With the *fit* in the regression, the Sharpe ratio for the HML premium is $1.58/\sqrt{378} = 0.081$, and that for the premium, $\gamma_4(\mathit{fit})$, is $7.8/\sqrt{378} = 0.401$. Applying MacKinlay's interpretation here suggests that if we accept the FF three-factor as a model for both expected returns and risk control, then the portfolio strategy implied by the *fit* is an attractive, near-arbitrage opportunity. Alternatively, we interpret the evidence as a striking rejection of the FF three-factor model.

E. Are These "Useless" Factors?

Although the results of the cross-sectional regressions are striking, they should be interpreted with some caution. Kan and Zhang (1999) provide an analysis of bias in cross-sectional regressions when there is a "useless" factor that has a true beta in time series equal to zero. They show that such a useless factor beta may appear with a large t-ratio in a cross-sectional regression, as the design matrix of the regression is ill-conditioned. Jagannathan and Wang (1998) provide an asymptotic analysis that includes a useless factor as a special case, and Ahn and Gadarowski (1998) extend their results with more general assumptions about heteroskedasticity and autocorrelation. Given that the lagged instruments have relatively small R-squares in the time-series, it is possible that our results reflect a bias as described by these studies.

Kan and Zhang (1999) suggest using the stability of cross-sectional coefficients in subperiods as a diagnostic tool to indicate the useless factor bias, as the cross-sectional coefficients should be unstable in the presence of a useless factor. Our rolling estimators provide an opportunity to look for instability. We examine time-series plots of our cross-sectional coefficients. Figure 1 shows an example. The cross-sectional regression coefficients on the *fit* are graphed over time. Superimposed on the graph are the monthly coefficients for the betas on the market index, a factor that is as far from useless in the time-series regressions as we can imagine. Since the units of the regressors—market beta versus *fit*—are different, we multiply the coefficient on the *fit* by the ratio of the time series means of the coefficient

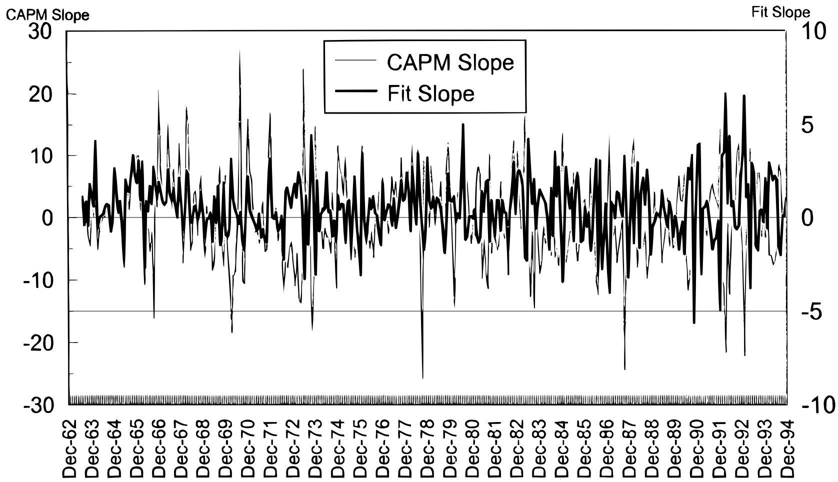

Figure 1. Comparison of cross-sectional slopes. The CAPM slope (left scale) shows the time series of monthly cross-sectional regression coefficients of size and book/market portfolio returns on their stock market betas. The fit slope (right scale) shows the time series of monthly cross-sectional regression coefficients of the portfolio returns on their *fit* values, scaled to have the same sample mean as the CAPM slopes. The figure illustrates that the fit slopes appear to be more stable over time than the CAPM slopes.

values. Scaled to the same means, the volatilities of the two time series are very different. The coefficients on the *fit* appear much more stable than those for the market beta. Indeed, to see the variation in both series on the same graph we use different scales: The *fit* coefficient is shown at a smaller scale than the market beta coefficient. Given this striking evidence, we do not believe that a useless factor story explains our results.

F. Results Using Efficient-Weighted Fama–MacBeth Regressions

Table VI summarizes cross-sectional regression results using the efficient-weighted version of the Fama–MacBeth coefficients, as derived in Appendix A. These essentially weight the coefficient each month in inverse proportion to the variance of the estimator from that month. A t-ratio for each coefficient is constructed similarly to Fama and MacBeth (1973), but the months are weighted to reflect the weighted estimator.

The results in Table VI confirm the finding that the *fit* allows us to reject the FF model in cross-sectional regressions. In three of the four cases, the *fit* t-ratio is significant given the FF factor loadings. Although the magnitudes should be interpreted with caution, as explained before, many of the patterns in the regression results are similar to those of Table V. Only in one of four cases does the coefficient on the HML loading produce a significant t-ratio when the fit is in the regression, and in no case is SMB significant. However, unlike the previous tables, the weighted average slope coefficient for HML is larger when the fit is in the regression.

Table VI

Efficient-Weighted Fama–MacBeth Regression Results

The efficient weighted average of the coefficients from monthly cross-sectional regressions are expressed as percentage per month, as derived in Appendix A. The dependent variables at time t are 25 value-weighted portfolio returns formed on size and the ratio of book-to-market, and measured in excess of the return on a 30-day Treasury bill. The regressors are a constant, the betas on the three FF factors, and a fitted conditional expected return estimated with data up to time $t-1$. The three FF factors are the market return (mkt), a small minus large market capitalization portfolio (smb), and a high minus a low book-to-market portfolio (hml). The betas are from a time-series regression of the portfolio excess returns on the excess factor returns using data to time $t-1$. The fitted expected returns (*fit*) are from time-series regressions of the returns on lagged instrumental variables, using data to time $t-1$. The lagged instrumental variables are described in Table II. The sample is July 1963 to December 1994 and the number of time-series observations is 378. The number of cross-sectional regressions is 377. For the first 60 months we use the in-sample betas. After observation 60, the sample for estimating the beta grows by one observation in Panel A. In Panel B, the regressions use a 60-month rolling window to estimate the betas (the time-series predicted returns use an expanding sample). t-statistics are reported under the average coefficients. γ_0 is the weighted-average intercept. The overall R^2 is derived in Appendix A.

γ_0	γ_1(mkt)	γ_2(smb)	γ_3(hml)	γ_4(*fit*)	Overall R^2
Panel A. With Expanding Sample Betas					
−0.073	0.453	0.796	0.275	—	0.0020
−0.123	0.601	1.990	1.442	—	—
0.410	—	—	—	1.912	0.0938
1.706	—	—	—	3.015	—
0.282	0.546	0.247	0.320	0.609	0.0946
0.480	0.734	0.627	1.616	0.275	—
Panel B. With 60-Period Rolling Sample Betas					
0.051	−0.527	0.999	0.047	—	0.0025
0.052	−0.478	1.789	0.180	—	—
0.208	—	—	—	1.333	0.0938
0.822	—	—	—	2.022	—
−0.154	−0.897	0.504	0.171	6.032	0.0941
−0.171	−0.889	0.988	0.627	1.405	—
Panel C. With Expanding Sample Conditional Betas					
−0.054	0.231	−0.341	−0.466	—	0.0042
−0.121	0.541	−1.032	−1.700	—	—
0.350	—	—	—	1.806	0.0938
1.472	—	—	—	2.960	—
−0.092	0.048	−0.409	−0.650	3.841	0.0950
−0.215	0.113	−1.231	−2.187	2.683	—
Panel D. With 60-Period Rolling Sample Conditional Betas					
0.328	0.308	0.690	−0.019	—	0.0018
0.762	0.831	0.949	−0.069	—	—
0.207	—	—	—	1.400	0.0938
0.815	—	—	—	2.095	—
0.147	0.064	0.393	−0.079	2.321	0.0944
0.426	0.178	0.760	−0.304	1.975	—

We observed earlier that the increments to time-series regression R-squares, for the portfolio returns regressed on the contemporaneous factors, are small when the lagged instruments are included in the regressions. Table VI includes estimates of overall R-squares, as derived in Appendix A. The overall R-squares combine the time-series and cross-sectional dimensions of model explanatory power, where each return-month is weighted inversely to its variance. For the FF model, the R-squares vary from 0.2 to 0.42 percent across the experiments. These figures are much lower than the cross-sectional regression R-squares reported in previous studies, reflecting the relatively poor fit of the FF three-factor betas to the time-series of the expected returns. (Recall that the explanatory variables are predetermined betas, not the contemporaneous factor values.) When the predetermined value of the fit is in the regressions, the R-squares range from 9.3 percent to 9.5 percent. These figures are similar to those obtained from time-series regressions of returns on the lagged instruments themselves. The comparison shows that the fit provides a dramatic improvement in the overall explanatory power, illustrating that the FF three-factor model is strongly rejected.

G. Digging deeper

Given that the time-series instruments deliver such a powerful cross-sectional predictor of stock returns, it is interesting to know which of the lagged variables are relatively important in the cross-sectional regressions. We repeat the cross-sectional analysis of the preceding section, replacing the fitted expected returns with the estimated regression coefficient, δ, on a single lagged instrument, and we study the instruments one at a time in the presence of the FF three-factor betas. The results are on the Internet.

The cross-sectional coefficients on the individual δ's show a number of interesting patterns. No individual coefficient drives the cross-sectional explanatory power. However, the coefficients for the lagged excess return of the three-month bill, δ_{HB3}, and for the lagged one-month yield δ_{Tbill}, are consistently strongly significant cross-sectional predictors. For example, the t-ratios for the slope coefficient for δ_{HB3} are between 2.6 and 3.8 in all of the 48 different specifications we examine. For δ_{Tbill} the t-ratios are all between 2.1 and 4.1. This suggests that the FF three-factor model leaves out important patterns in expected stock returns that are related to cross-sectional differences in the portfolios' sensitivity to lagged interest rates.

H. Tests on a Four-Factor Model

The idea that the FF factor model may leave out important interest rate exposures is reflected in the work of Elton, Gruber, and Blake (EGB, 1995), who advocate a four-factor model. The first three factors are similar to those of the FF model, and the fourth factor is a low-grade bond portfolio excess return. We repeat the battery of tests described above using the EGB four-factor model as the null hypothesis, with data over the February 1979 to

December 1993 period, a total of 180 monthly observations.[13] The main results are summarized here, and are available by request or on the internet.

When we test for time-varying betas of the size- and book/market-sorted portfolios, as in Table III, we find evidence of time-varying betas in the four-factor model. The F-tests produce 10 out of 25 p-values less than 0.05, and the Bonferroni inequality implies that the p-value of a joint test across the 25 portfolios is less than 0.001. There is also evidence of time-varying alphas in this model, similar to Table IV. As a prelude to the cross-sectional regressions we examine the average cross-sectional correlations of the four-factor beta estimates and we find no strong correlations. This suggests that the $(x'x)$ matrix in the cross-sectional regressions should not be ill-conditioned due to colinearity of the regressors.

The cross-sectional regression analysis, similarly to Table V, reveals some interesting results for the four-factor model. In the presence of the bond-return factor, the betas on the EGB market, size, and value-growth factors are seldom individually significant in the cross-sectional regressions. By itself, the fitted expected return produces t-ratios between 3.8 and 5.8 in experiments corresponding to the eight panels of Table V. When the four-factor betas and the *fit* are in the regression, the t-ratios for the *fit* are between 3.3 and 5.6. No four-factor beta is individually significant in the presence of the *fit*.

In summary, the results for the EGB four-factor model are similar to the results for the FF three-factor model. Conditional on the lagged instruments the alphas in either model are time-varying and thus not zero. This implies that the models do not explain the conditional expected returns of these portfolios. Even conditional versions of the models, with time-varying betas, do not capture the dynamic patterns of the expected returns. The lagged instruments do not explain much the time-series variance of the returns. However, in cross-sectional regressions the *fit* is a relatively powerful regressor. Its Fama–MacBeth t-ratios are large even with the factor betas in the regression.

IV. Interpreting the Evidence

The preceding evidence shows that variables used to proxy for expected returns over time in the conditional asset pricing literature also provide a potent challenge for the Fama–French and Elton–Gruber–Blake variables in explaining the cross-section of conditional expected returns. These results carry implications for risk analysis in market efficiency studies, performance measurement, cost-of-capital calculations, and other applications.

Factor models are frequently used to control for risk in studies of market efficiency. This is typically done by regressing returns on the factors and taking the residuals, perhaps added to the intercept, as a measure of risk-adjusted returns. Alternatively, returns may be measured in excess of the return on a matching portfolio, constructed to have similar market capital-

[13] We are grateful to Chris Blake for providing data on the EGB factors.

ization and book/market ratio as the firm to be studied. Such an approach is required in a situation such as a study of initial public offerings (IPOs), as no prior returns data are available to estimate a regression model. If size and book/market are good proxies for risk, then the matching portfolio provides a risk adjustment. Our evidence casts serious doubt on the empirical validity of such a procedure. Matching the market, small-firm, and book/market exposure is expected to leave predictable dynamic behavior in the "risk-adjusted" returns. When studying the performance of portfolios based on a phenomenon that is correlated with aggregate economic activity, such as IPOs, the risk of falsely detecting "market inefficiencies" is likely to be especially acute. This is because the lagged instruments are likely to be correlated with the event in question.

Another recent application of the FF and EGB factor models is in measuring the performance of mutual funds. Here, a regression of the fund on the factor excess returns produces an intercept that is interpreted as a multi-beta version of Jensen's (1968) alpha. However, our evidence shows that even the hypothetical, mechanically constructed portfolios in our study have nonzero alphas in these models. The alphas are time-varying and can be modeled as simple functions of publicly available, lagged instruments. Since these portfolios can in principle be traded and the instruments are known, it should be a simple matter for a fund to "game" a performance measure constructed using these models. From this perspective, the performance of funds in relation to such strategies remains an open puzzle.[14]

Factor models for expected returns, and the CAPM in particular, have long been used in corporate cost-of-capital calculations. Here, the idea is to find an expected return commensurate with the risk of a project, and to discount prospective cash flows at the risk-adjusted return to determine its present value. Studies such as Fama and French (1997) have put the FF factor model to this application, and some have used it in practice. Of course, the lack of theoretical grounding for the FF model is a serious limitation in this context. For example, taken literally the model suggests that a firm could change its capital costs by altering its book value, other things equal. Our empirical evidence provides additional reasons to be suspicious of the FF model as a source of risk-adjusted discount rates.

Our empirical results may also be interpreted from a technical perspective, in view of portfolio efficiency. A portfolio is minimum-variance efficient if and only if expected returns in the cross section are a linear function of asset's covariances with the portfolio return (e.g., Roll (1977)). If betas on the FF factors provide a reasonable description of the cross section of the unconditional expected returns of these portfolios, then a combination of the factors is a fixed-weight, unconditionally efficient portfolio. If the lagged

[14] Becker et al. (1999) find that, although hypothetical portfolios of value stocks return more than growth stocks, portfolios of value-investing mutual funds grouped on similar criteria in their equity holdings do not offer higher returns than growth mutual funds. The difference is not explained by higher expense ratios for growth funds.

variables deliver a good proxy for the conditional expected returns at each date, given the lagged instruments Z_t, the *fit* is proportional in the cross section to betas on a *conditional* minimum-variance portfolio given Z_t. The Fama–MacBeth regressions use the actual future returns each month as the dependent variable. These may be viewed as equal to the unconditional expected returns plus noise, or as equal to the conditional expected returns plus a smaller-variance noise. The covariances with a conditionally efficient portfolio should therefore provide a more powerful regressor in the Fama–MacBeth approach, with smaller errors than would the covariances with an unconditionally efficient, fixed-weight portfolio.[15]

Although the portfolio efficiency interpretation of our results does not require a risk-based asset pricing model, if a risk-based model determines expected returns then the results carry implications about the model. These may provide direction for future research attempting to identify better-specified asset pricing models. In a risk-based asset pricing model, expected excess returns are proportional to securities covariances with a marginal utility of wealth. In essence, we should be looking for models in which the cross section of the conditional covariances with the marginal utility captures the cross section of the *fit*.

V. Robustness of the Results

We conduct a number of additional experiments to assess the sensitivity of our results to the portfolio grouping procedures and the empirical methods. The results of these experiments are described in this section. Tables of these results are available by request, or on the Internet.

A. Errors-in-Variables

The cross-sectional regressions are likely to be affected by errors-in-variables when the first-pass time-series regression coefficients appear on the right-hand side. If the factors are measured with error, we may falsely reject a model by introducing an explanatory variable that is correlated with the true factor betas. Kim (1997) explores the possibility that the CAPM is rejected by a book-to-market factor for this reason, and we cannot rule out a similar explanation for our rejections of the FF model. Since it is not clear what risks the FF factors may represent, it is hard to consider measuring those factors without error.

Errors in variables arise even when the first-pass regressions are unbiased, as a result of the sampling error in the first-pass estimator. This is the classic generated regressor problem, known to bias the second-pass, cross-

[15] We emphasize that the unconditional efficiency is defined here within the set of fixed-weight portfolios of the test assets. This is to distinguish from the notion of unconditional efficiency in Hansen and Richard (1987), which is defined over the set of all dynamic trading strategies that may depend on the conditioning information. See Bansal and Harvey (1997) and Ferson and Siegel (1997) for treatments of efficiency with dynamic trading strategies.

sectional regression slopes in finite samples and their standard errors even in infinite samples (see Pagan (1984), Shanken (1992), Kim (1995, 1997), and Kan and Zhang (1999) for recent analyses). The first-pass regression coefficients may also be biased in finite samples even without measurement errors in the factors (e.g., Stambaugh (1998), Kothari and Shanken (1997)).

Though measurement error problems are potentially complex, they are likely to be more severe in the time-series coefficients of the *fit* than in the estimates of the FF factor betas, because the explanatory power of a time-series regression on the contemporaneous FF factors is much higher than on the lagged instruments. Errors-in-variables therefore probably works against our ability to find that the lagged instruments are significant, suggesting that our results are conservative in view of measurement error. However, when there is correlated measurement error in a multiple regression the direction of the effect may be difficult to predict. We wish to be conservative about our evidence that the *fit* rejects the FF model. Therefore, we conduct experiments to assess the likely robustness of our results to measurement errors.

We repeat our analysis using the actual values of size and book/market in place of time-series betas on the FF factor-portfolios. As these attributes are likely to be measured more precisely than the time-series regression coefficients, this skews the measurement error further in favor of the FF model. We use data on 25 portfolios, sorted on the basis of book/market and size, together with the actual values of the log of the market capitalization (lnSize) and the log of the book/market ratio (lnB/M) measured similarly to Fama and French (1992).[16] The data cover the July 1964 to December 1992 period, a total of 342 observations.

We repeat our previous tests for time-varying betas and alphas using this slightly different sample of returns, and the results are similar to those reported above. We find strong evidence of time-varying betas and alphas. Table VII focuses on the cross-sectional regressions, similar to those in Table V but using the actual lagged values of the attributes instead of the FF betas for SMB and HML. When the market betas, lnSize and lnB/M are used alone in the regressions, the results are as expected from Fama and French (1992). When the *fit* is included in the cross-sectional regressions, its *t*-ratios are 4.3 or larger in every case we consider. This is striking evidence against the FF three-factor model, especially in view of the measurement error issue.

As an additional check, we run cross-sectional regressions using betas on the FF factors and on the time-series of the fitted cross-sectional coefficients obtained from Table V, treating the latter as competing excess returns or “factors.” This approach should place the *fit* at a further measurement error disadvantage, relative to the FF factors. We find that the *fit* loadings produce a Fama–MacBeth *t*-ratio larger than 1.95 in three of the four panels corresponding to Table V.

[16] These data are courtesy of Raymond Kan and Chu Zhang, to whom we are grateful. The sorting criteria are somewhat different than in our first sample; see Appendix B for details.

Table VII

Attributes and the Cross Section of Stock Returns

The average coefficients from monthly cross-sectional regressions are expressed as percentage per month. The dependent variables at time t are 25 value-weighted portfolios formed on size and the book/market ratio, and measured in excess of the return on a 30-day Treasury bill. The regressors are a constant, the portfolios' betas on a stock market factor, the portfolio size (natural log of market capitalization, lnSize), the log of the book/market ratio (ln(B/M)), and a fitted conditional expected return estimated with data up to time $t-1$ (*fit*). The market betas are from time-series regressions of the returns on the excess market factor return using data to time $t-1$. The fitted expected return is from a time-series regression of the portfolio returns on lagged instrumentals using data to time $t-1$. The lagged instrumental variables are described in Table II. t-statistics are reported under the average coefficients. The sample is August 1964 to December 1992, the number of time-series observations is 342, and the number of cross-sectional regressions is 341. For the first 60 months we use the in-sample betas. After observation 60, the sample for estimating the beta grows by one observation in Panel A. In Panel B, the regressions use a 60-month rolling window to estimate the market betas (the time-series predicted returns use an expanding sample).

γ_0	$\gamma(\beta_{\text{mkt}})$	γ(lnSize)	γ(ln(B/M))	γ(*fit*)
Panel A. With Expanding Sample Betas				
1.491	−0.017	−0.134	0.226	—
2.906	−0.043	−2.619	2.282	—
0.337	—	—	—	0.506
1.177	—	—	—	4.967
0.598	0.226	−0.063	0.188	0.308
1.242	0.547	−1.289	1.924	4.300
Panel B. With 60-Period Rolling Sample Betas				
1.588	−0.217	−0.119	0.240	—
3.331	−0.677	−2.393	2.476	—
0.337	—	—	—	0.506
1.177	—	—	—	4.967
0.695	0.010	−0.050	0.199	0.328
1.536	0.032	−1.060	2.072	4.684
Panel C. With Expanding Sample Conditional Betas				
1.479	0.012	−0.138	0.222	—
3.110	0.044	−2.759	2.287	—
0.337	—	—	—	0.506
1.177	—	—	—	4.967
0.770	−0.025	−0.056	0.199	0.324
1.686	−0.091	−1.187	2.065	4.663
Panel D. With 60-Period Rolling Sample Conditional Betas				
1.480	0.046	−0.141	0.224	—
3.308	0.326	−2.766	2.299	—
0.337	—	—	—	0.506
1.177	—	—	—	4.967
0.834	0.042	−0.069	0.186	0.336
1.889	0.296	−1.480	1.910	4.529

Although these additional experiments increase our confidence that our results are robust to measurement errors, it seems impossible to completely resolve the measurement error issue without knowledge of the underlying "true" model of expected returns.

B. Results for Industry Portfolios

We replicate the tests of the previous sections using a sample of industry portfolio returns. The data are from Harvey and Kirby (1996) and are described in Appendix B. Industry portfolios are interesting in view of the evidence in Fama and French (1997), who use the FF three-factor model to estimate industry costs of capital. Since the FF factors are designed to explain the returns on size and book/market portfolios, we expect them to perform less well on portfolios grouped by alternative criteria.

We find strong evidence that the lagged market indicators enter as instruments for time-varying betas on the industry portfolios. The F-tests for 22 of the 25 portfolios produce p-values below 0.05, and a joint Bonferroni test strongly rejects the hypothesis that the three-factor betas are constant. Compared with our tests in Table III, this is consistent with the observation of Fama and French (1997) that the betas of industries vary over time more dramatically than portfolios sorted on size and book/market.

The portfolio excess returns are regressed on a constant and the three FF factors, and a t-test is conducted for the hypothesis that the intercept is equal to zero. Like the size and book/market portfolios, this test produces little evidence against the hypothesis that the FF variables can unconditionally price the 25 industry portfolios and fixed-weight combinations of their returns; only five of 25 p-values are less than 0.05.

We regress the portfolio excess returns on the three FF factors and the vector of lagged instruments. The F-test for the hypothesis that the vector of instruments may be excluded from the regression produces 25 p-values; all are less than 0.01. When we allow for both time-varying betas and time-varying alphas and test the hypothesis that the alphas are constant, we find 24 of the 25 p-values are below 0.01. In summary, the industry portfolio evidence against the FF three-factor model is even more striking than is the evidence based on the book/market portfolios.

We repeat our tests of the EGB four-factor model using the industry portfolios in place of the size- and book/market-sorted portfolios. We find slightly weaker evidence of time-varying betas and alphas here than in the other portfolio design. Still, the tests reject the hypotheses of constant betas or alphas. The cross-sectional regression analysis produces results generally similar to those described earlier.

C. Size, Book-to-Market, and Momentum Portfolios

Fama and French (1996) found that their three-factor model was most seriously challenged by the "momentum" anomaly described by Jegadeesh and Titman (1993). This is the observation that portfolios of stocks with relatively high returns over the past year tend to have high future returns. To see if our results are sensitive to portfolios grouped on momentum, we obtain data from Carhart, et al. (1996).[17] In each month t, Carhart et al. (1996) group the common stocks on the CRSP tape into thirds according to three independent criteria, producing 27 individual portfolio return series. The grouping criteria are (1) market equity capitalization, (2) the ratio of book equity to market equity, and (3) the past return for months $t-2$ to $t-12$. The data are available for the same sample period as our previous analysis, so we can conduct a controlled experiment by using the same lagged instrument data.

Conducting the tests for time-varying betas as in Table III, we find strong evidence that the betas on the FF factors vary with the lagged instruments. The largest of the 27 p-values from the F-tests is 0.029. Examining the alphas as in Table IV, we find that the unconditional alphas are larger than in the original 25 portfolios, consistent with the findings of Fama and French (1996). They are as large as -11 percent per year. Testing for zero unconditional alphas using F tests, 16 of the 27 p-values are less than 0.05 and the Bonferroni p-value is less than 0.001. Testing for constant alphas in conditional models with time-varying betas, the largest of the 27 p-values is less than 0.001.

We examine cross-sectional regressions and find, similarly to Table V, that the results are consistent with those using the other portfolio designs. When the fitted conditional expected return is used alone in the cross-sectional regressions, its t-ratio varies between 7.9 and 8.3. When all four variables are used, the t-ratio for fitted expected return remains strong, between 7.5 and 8.4.

D. Data Mining

The issue of data mining has been raised in previous studies, both in connection with the size and book/market effects in the cross section of stock returns and in connection with the lagged instruments in the time series of returns (e.g., Lo and MacKinlay (1990), Black (1993), Breen and Korajczyk (1994), Foster, Smith, and Whaley (1997)). With data mining, a chance correlation in the data may be "discovered" to be an interesting economic phenomenon. An empirical regularity that is dredged from the data by chance is not expected to hold up outside of the sample that generated it. Since many researchers use the same data in asset pricing studies, a collective form of data mining is a severe risk. Of particular concern here is the extent to which our results may be an artifact of data mining.

Although we can not rule out a potential data mining bias in our results, we have reasons to suspect this is not a serious problem. There have been out-of-sample studies that help to mitigate concerns about data mining in

[17] These data are courtesy of Mark Carhart, to whom we are grateful.

the cross-sectional analysis of book/market. For example, Chan, Hamao, and Lakonishok (1991) and Fama and French (1998) find book/market effects in the cross section of average returns in Japan and other countries. Davis, Fama, and French (1998) extend the results in U.S. data back to 1929. Barber and Lyon (1997) find the effects in a sample of U.S. firms that were not used by Fama and French in their original (1992) study.

There is also out-of-sample evidence that helps to mitigate concerns about data mining in the time-series predictive ability of the lagged instruments. The lagged Treasury bill rate, for example, was noted by Fama and Schwert (1977). If its explanatory power was a statistical fluke, it should not have remained a potent predictor, as it has, in more recent samples. Pesaran and Timmerman (1995) present an analysis of the ability of a set of lagged instruments, similar to ours, to predict returns in periods after they were discovered and promoted in academic studies.

We have an additional reason to believe that our results are not an artifact of data mining. Even if the lagged instruments are dredged from the data in previous studies, they are selected primarily for their ability to predict stock returns over time. We can think of no reason that a spurious time-series correlation with returns should produce a spurious ability to explain the cross-section of portfolio returns.

VI. Concluding Remarks

Previous studies identify predetermined variables with some power to explain the time series of stock and bond returns. This paper shows that loadings on the same variables also provide significant cross-sectional explanatory power for stock portfolio returns. We use time-series loadings on the lagged variables to conduct powerful tests of empirical models for the cross section of stock returns. We reject the three-factor model advocated by Fama and French (1993) even in a sample of equity portfolios similar to the one used to derive their factors. We also reject the four-factor model advocated by Elton, Gruber, and Blake (1995). The results are robust to variations in the empirical methods, and to a variety of portfolio grouping procedures.

Our focus is not to search for alternatives to the factors advocated by Fama and French and Elton, Gruber, and Blake. Our evidence does suggest that applications of these factor models should control for time-varying betas, and that doing so provides some improvement. However, even conditional versions of the models, with time-varying betas, appear to leave significant predictable patterns in their pricing errors.

Loadings on lagged instruments reveal information that is not captured by these popular factors for the cross section of expected returns. This should raise a caution flag for researchers who would use the FF or EGB factors in an attempt to control for systematic patterns in risk and expected return. The results carry implications for risk analysis, performance measurement, cost-of-capital calculations, and other applications.

Appendix A

A. *Efficient Weighting of Fama–MacBeth Regressions*

Consider a pooled time-series and cross-section regression model written similarly to Litzenberger and Ramaswamy (1979), as:

$$Y = X\gamma + U, \qquad E(UU') = \Omega, \tag{A1}$$

where Y is a $TN \times 1$ vector. The first N rows are the returns of N stock portfolios for the first month of the sample, followed by the second month, and so on. There are T months in the sample. The $TN \times K$ matrix X has a column of ones, and the remaining columns are the predetermined portfolio attributes, such as the betas, book-to-market ratios, or the fitted expected returns, stacked up like the dependent variable. The $K \times 1$ vector of parameters, γ , are the average risk premiums that we wish to estimate. The $TN \times TN$ covariance matrix is Ω.

Under standard assumptions the generalized least squares estimator is best linear unbiased and is given as:

$$\gamma_{\mathrm{GLS}} = (X'\Omega^{-1}X)^{-1}X'^{-1}Y. \tag{A2}$$

We make the assumption that the error terms are uncorrelated over time but correlated across stock portfolios with a general $N \times N$ covariance matrix at date t, Ω_t. This implies that Ω has a block diagonal structure with the Ω_t's on the diagonal. Using this structure in equation (A2), the GLS estimator may be written as:

$$\gamma_{\mathrm{GLS}} = (\Sigma_t X_t'\Omega_t^{-1}X_t)^{-1}(\Sigma_t X_t'\Omega_t^{-1}Y_t), \tag{A3}$$

where Σ_t indicates summation over time. Now, the GLS version of the Fama–MacBeth coefficient for month t may be written as

$$\gamma_{FM,t} = (X_t'\Omega_t^{-1}X_t)^{-1}(X_t'\Omega_t^{-1}Y_t). \tag{A4}$$

From equations (A3) and (A4) we can express the full GLS estimator as:

$$\gamma_{\mathrm{GLS}} = \Sigma_t\{(\Sigma_t X_t'\Omega_t^{-1}X_t)^{-1}(X_t'\Omega_t^{-1}X_t)\}\gamma_{FM,t}, \tag{A5}$$

which shows that the efficient GLS estimator is a weighted average of the Fama–MacBeth estimates with the weights for each date t proportional to $X_t'\Omega_t^{-1}X_t$.

From equation (A4) we calculate the variance of a typical Fama–MacBeth estimator for month t as $E\{(\gamma_{FM,t} - \gamma)(\gamma_{FM,t} - \gamma)'\} = (X_t'\Omega_t^{-1}X_t)^{-1}$. Thus, we can see that the efficient weighting of the FM estimators in equation (A5) places more weight on the months with lower variance estimators, and less weight on a month with an imprecise estimate.

The standard errors of the GLS estimates may be obtained from the usual expression: $Var(\gamma_{\mathrm{GLS}}) = (\Sigma_t X_t'\Omega_t^{-1}X_t)^{-1}$. However, when N is large relative to T (e.g., a standard design with a rolling regression estimator of beta, N=25, and T=60), full covariance GLS is not practical. In such cases weighted least squares (WLS) may be used, which assumes that Ω_t is diagonal. But with a diagonal covariance matrix the standard error estimator does not capture the strong cross-sectional dependence in stock returns, which motivates the original Fama–MacBeth approach.

Fama and MacBeth (1973) suggest calculating a standard error for the overall coefficient from the time-series of the monthly estimates. The variance of the sample mean of the monthly estimates is $(1/T)(T^{-1}\Sigma_t\gamma_{FM,t}^2 - (T^{-1}\Sigma_t\gamma_{FM,t})^2)$, which assumes that the model errors are uncorrelated over time but cross-sectionally dependent.

We provide a simple modification of the approach of Fama and MacBeth for the efficient-weighted FM estimator. We first express $\gamma_{GLS} = \Sigma_t w_t\gamma_{FM,t}$, where the weight for each month, $w_t = ((\Sigma_t X_t'\Omega_t^{-1}X_t)^{-1}(X_t'\Omega_t^{-1}X_t))$. The variance may be obtained as

$$s^2(\gamma_{\mathrm{GLS}}) = (1/T)(T^{-1}\Sigma_t w_t^2\gamma_{FM,t}^2 - (T^{-1}\Sigma_t w_t\gamma_{FM,t})^2). \tag{A6}$$

The standard errors for the efficient-weighted FM estimator are thus obtained by replacing $\gamma_{FM,t}$ by $w_t\gamma_{FM,t}$ in the usual calculation.

B. A Measure of Explanatory Power

The simplest measure of explanatory power in a regression model is the coefficient of determination, or R-squared. However, the usual R-squared is difficult to interpret in a cross-sectional regression for stock returns because of the strong cross-sectional dependence. Consider a standard, GLS-transformed version of equation (A1):

$$\tilde{Y} = \tilde{X}\gamma + \tilde{U}, \qquad E(\tilde{U}\tilde{U}') = I_{TN}, \tag{A7}$$

where $\tilde{Y} = \Omega^{-1/2}Y$, $\tilde{X} = \Omega^{-1/2}X$, and $\tilde{U} = \Omega^{-1/2}U$. In the transformed model there is no time-series or cross-sectional correlation of the errors, and the errors are homoskedastic. We use the R-squared of the transformed model as a measure of the overall explanatory power. The GLS R-squared is advocated by Kan and Zhang (1999) for cross-sectional regressions. The overall measure here gives equal weight to the time-series and cross-sectional dimensions of explanatory power in the transformed model. Within a given cross section, observations with larger standard deviations are given smaller weight. In the time-series dimension, months with larger standard deviations of the error term are given smaller weights.

Define demeaned variables, $y_{it} = Y_{it} - N^{-1}T^{-1}\Sigma_t\Sigma_i Y_{it}$, demeaned using the grand mean, taken over both the time series and cross section. Stack the y_{it}'s into a $TN \times 1$ vector, y, using the same convention as before. The demeaned predictors x and the residuals, u, are defined analogously. A simple expression for the overall R-square measure uses the $TN \times 1$ vectors y, x, and u. The R-square for the transformed model (A7) is $1 - (u'\Omega^{-1}u)/(y'\Omega^{-1}y)$. Substituting the expression for γ_{GLS} with the assumed diagonal structure of Ω, we can express the R-square in terms of the demeaned N-vectors of the original data:

$$R^2 = (\Sigma_t y_t' \Omega_t^{-1} x_t)(\Sigma_t x_t' \Omega_t^{-1} x_t)^{-1}(\Sigma_t x_t' \Omega_t^{-1} y_t)/(\Sigma_t y_t' \Omega_t^{-1} y_t). \tag{A8}$$

In a typical application such as ours, full covariance GLS is not practical. We therefore use a weighted least squares version of equation (A8). We replace Ω_t with a diagonal matrix using an estimate of the variance of the residuals for each test asset in a given month on the diagonals.

Appendix B

A. Book-to-Market and Size-Sorted Portfolios

Returns on 25 value-weighted portfolios of the common stock of firms listed on the New York Stock Exchange (NYSE) and covered by COMPUSTAT are formed. Following Dimension Fund Advisors' exclusion criteria, foreign firms, ADRs, and REITs are excluded. Portfolios are formed by ranking firms on their market capitalization (size) in June of each year and the ratio of book value to market value of equity (BE/ME) as of December of the preceding year. The size and BE/ME sorts are independent. Firms are ranked and sorted annually into five groups. Monthly portfolio returns are then computed from July of year $t+1$ to June of $t+2$ for each group. BE is Stockholder's Equity (A216) less Preferred Stock Redemption Value (A56) (or Liquidating Value (A10), or Par Value (A130), depending on availability), plus balance sheet deferred taxes (A35), if available. If Stockholders Equity is not available, it is calculated as Total Common Equity (A60) plus the par value of preferred stock (A130).

B. Industry Portfolios

Monthly returns on 25 portfolios of common stocks are from Harvey and Kirby (1996). The portfolios are value-weighted within each industry group. The industries and their SIC codes are listed in Table BI.

Table BI

Standard Industrial Classifications for Industry Portfolios

Number	Industry	SIC Codes
1	Aerospace	372, 376
2	Transportation	40, 45
3	Banking	60
4	Building Materials	24, 32
5	Chemicals/Plastics	281, 282, 286–289, 308
6	Construction	15–17
7	Entertainment	365, 483, 484, 78
8	Food/Beverages	20
9	Healthcare	283, 284, 385, 80
10	Industrial Machinery	351–356
11	Insurance/Real Estate	63–65
12	Investments	62, 67
13	Metals	33
14	Mining	10, 12, 14
15	Motor Vehicles	371, 551, 552
16	Paper	26
17	Petroleum	13, 29
18	Printing/Publishing	27
19	Professional Services	73, 87
20	Retailing	53, 56, 57, 59
21	Semiconductors	357, 367
22	Telecommunications	366, 381, 481, 482, 489
23	Textiles/Apparel	22, 23
24	Utilities	49
25	Wholesaling	50, 51

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